Accuracy Limits of pQCD at Low Energy, below 10 GeV

Dmitry SHIRKOV

Bogoliubov Lab, JINR Dubna

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[a] CALC '06, Dubna

Topics

1. Ambiguity of the PT beta function and QCD coupling
2. **Denominator representation** for α_s vs canonical expression
3. The π^2- terms in Minkowskian coupling $\alpha_s(Q^2) \to \alpha_M(s)$
4. Ghost free Analytic Perturbation Theory :
   - Ghost free Euclidean $\alpha_E(Q^2)$ and Minkowskian $\alpha_M(s)$ couplings
   - $\int$ transformations Euclidean $\Leftrightarrow$ Minkowskian
   - QCD observables via **Non-power sets** $\{\mathcal{A}_k\}$, $\{\mathfrak{A}_k\}$
5. Simple analytic **Models** for APT 3-loop functions
6. The issue of **Scale Arbitrariness**
7. Limits of accuracy of perturbative QCD theory

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Preamble, I

Claim 1 : Instead of canonical "3-storey" PDG eq.

$$\bar{\alpha}_s^{(3)}(x) = \frac{1}{\beta_0 L} - \frac{b_1}{\beta_0^2}\frac{\ln L}{L^2} + \frac{1}{\beta_0^3 L^3}\left[b_1^2(\ln^2 L - \ln L - 1) + b_2\right] -$$

$$-\frac{1}{\beta_0^4 L^4}\left[b_1^3\left(\ln^3 L - \frac{5}{2}\ln^2 L - 2\ln L + \frac{1}{2}\right) + 3b_1 b_2 \ln L - \frac{b_3}{2}\right] \quad (9.5-pdg)$$

use a more simple & precise "**Denominator expression**",

$$\bar{\alpha}_s^{(3,iter)}(L) = \frac{1}{\beta_0\, L + b_1\, \ln\left[L + \frac{b_1}{\beta_0}\, \ln L\right] + \frac{(b_1^2 - b_2)}{\beta_0\, L} + \frac{b_3}{2(\beta_0 L)^2}}; \quad (9.5 - den)$$

At the $f = 5\,;\, q \geq 5\mathrm{GeV}$ 2-loop (NLO) or 3-loop (NNLO) approx. (9.5-den) provides us with 2 or 1 per cent accuracy

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Preamble, II

Claim 2: Expansions for QCD observables in powers **of the same** $\bar{\alpha}_s$, i.e., in $[\bar{\alpha}_s(Q^2)]^k$ or $[\bar{\alpha}_s(s)]^k$, in Euclidean(Q^2) or Minkowskian(s) pictures, are not adequate practically to low-energy QCD.

Instead, in the Analytic Perturb Theory, one should use:

– Two different ghost-free couplings: $\alpha_E(Q^2)$, $\alpha_M(s)$

– Non-power functional expansions :

$$d(Q^2) = d_1 \alpha_E(Q^2) + d_2\,\mathcal{A}_2(Q^2) + d_3\,\mathcal{A}_3(Q^2) + \ldots$$

$$r(s) \;=\; d_1\,\alpha_M(s) \;+\; d_2\,\mathfrak{A}_2(s) \;+\; d_3\,\mathfrak{A}_3(s) \;+\; \ldots$$

sets $\{\mathcal{A}_k\}$, $\{\mathfrak{A}_k\}$ related $\mathcal{A}_k \leftrightarrow \mathfrak{A}_k$ by integral transform-ion.

This is essential below 10 GeV scale.

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Preamble, III

Claim 3: At modern level of data errors, the most important issue of QCD theory analysis is the **choice of the scale s** for effective coupling $\alpha_s(s)$, as the scale error, rather than higher-loops effects, limits the value of theoretical analysis.

This is essential for low energy region.

1. Arbitrariness of PT beta function $\beta(\alpha_s)$

2-loop beta function can be transformed to Padé-like form

$$\beta_2(\alpha_s) = -\beta_0\,\alpha_s^2 - \beta_1\,\alpha_s^3 \quad \to \quad \beta_2^P(\alpha_s) = -\frac{\beta_0\,\alpha^2}{1-b_1\,\alpha}\,; \;\; b_1 = \frac{\beta_1}{\beta_0}\,,$$

$$\beta_0 L = \frac{1}{\alpha_s^{(2)}} + b_1\,\ln\frac{\beta_0\,\alpha_s^{(2)}}{1+b_1\,\alpha_s^{(2)}}\,; \qquad \beta_0 L = \frac{1}{\alpha_s^{(2P)}} + b_1\,\ln(\beta_0\,\alpha_s^{(2P)})\,, \tag{1}$$

After usual iteration - 2 different forms

$$\alpha_s^{(2),iter} = \frac{1}{\beta_0 L + b_1 \ln(L + b_1/\beta_0)}\,; \qquad \alpha_s^{(2P),iter} = \frac{1}{\beta_0\,L + b_1 \ln L}\,. \tag{2}$$

Their relative difference

$$\left[\alpha_s^{(2P),iter} - \alpha_s^{(2),iter}\right]/\alpha_s^{2,iter} = \varepsilon_2(L) = \left(\frac{b_1}{\beta_0\,L}\right)^2,$$

is 5% at $q \sim 1\,\mathrm{GeV}$ and 1% at $q \sim 1.5\mathrm{GeV}$, $(n_f = 3)$.

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1. Ambiguity of PT beta function & α_s , II

Analogous analysis for 3-loop case reveals that

$$\left\{\alpha_s^{3,iter} - \alpha_s^{3P,iter}\right\} / \alpha_s^{3,iter} = \varepsilon_3 \sim \text{ (numerically !) } \varepsilon_2 \,.$$

is again 5% at $\mu \sim 1\,\mathrm{GeV}$ and 1% at $q \sim 1.5\,\mathrm{GeV}$.

This means that, generally, there is no such notion as "exact PT calculation", both analytic or numerical. We have no criterion for a choice between "normal" polynomial form for $\beta(\alpha)$ or the Padé-like one.

2. “Canonical” form of $\alpha_{\rm s}(\mu)$ vs “Denominator” one

Transition from "Denominator eq.",

$$\bar{\alpha}_s^{(3,iter)} = \frac{1}{\beta_0\, L + b_1\, \ln\left[L + \frac{b_1}{\beta_0}\, \ln L\right] + \frac{(b_1^2 - b_2)}{\beta_0\, L} + \frac{b_3}{2(\beta_0 L)^2}} \quad (9.5-den)$$

to “canonical" $1/L$-expanded form **(PDG, Bethke)** involves

$$\bar{\alpha}_s^{(3,can)}(x) = \frac{1}{\beta_0 L} - \frac{b_1}{\beta_0^2}\frac{\ln L}{L^2} + \frac{1}{\beta_0^3 L^3}\left[b_1^2(\ln^2 L - \ln L - 1) + b_2\right] -$$

$$-\frac{1}{\beta_0^4 L^4}\left[b_1^3\left(\ln^3 L - \tfrac{5}{2}\ln^2 L - 2\ln L + \tfrac{1}{2}\right) + 3b_1 b_2 \ln L - \tfrac{b_3}{2}\right] \qquad (9.5 - pdg)$$

casual parameter $\epsilon_{can}(q) = \frac{b_1 \ln L}{\beta_0 L} \sim$ 0.3-0.25 at $n_f = 3, 4.$
$\epsilon_{can}^2 \sim 0.1$, $\epsilon_{can}^3 \sim 0.02 - 0.03$ vs $\varepsilon_2 \sim \varepsilon_3 \sim 0.01$ at $1.5\,\mathrm{GeV}$.
Ghost trouble of (9.5-pdg), at $L = 0$ is much weaker in (9.5a). For solution of the ghost issue turn to
Analytic Perturbation Theory (APT)

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3. The π^2- terms in Minkowskian & Distance

Log's expansion $F(L) = 1 + \sum_{k\geq 1} f_k L^k\,;\ L_Q = \ln(Q^2/\mu^2)$ is slightly modified by integral transformations:

E.g., under Fourier an "Adler" transformations

$$\mathbb{F}_{\sin}[f](Q^2) \equiv \frac{2}{\pi}\int_0^\infty \frac{d\,r}{r}\,\sin(Qr)\,f(r^2) = \bar{f}(Q^2)\ .$$

$$\mathbb{D}[R](Q^2) \equiv Q^2\int_0^\infty \frac{ds}{(s+Q^2)^2}\,R(s)\ = D(Q^2)\,. \qquad (3)$$

$$L^2 \to L_s^2 - \pi^2\,; \quad L_Q^2 \to (L_r + \mathbf{C})^2 - \pi^2/12\,.$$

4. Main Results of APT. One-loop case.

For 1-loop case, APT formulae are simple. Starting with common RG-improved QCD coupling $\alpha_s^{(1)}(Q^2) = \frac{1}{(\beta_0\, l)}$, one arrives at ghost-free effective Euclidean

$$\mathcal{A}_1^{(1)}(l) = \frac{1}{\beta_0}\left(\frac{1}{l} - \frac{1}{e^l - 1}\right) = \frac{1}{\beta_0 \pi}\left(\frac{1}{l} + \frac{\Lambda^2}{\Lambda^2 - Q^2}\right), l = \ln\left(\frac{Q^2}{\Lambda^2}\right) \tag{4}$$

and Minkowskian

$$\mathfrak{A}_1^{(1)}(L) = \frac{1}{\beta_0 \pi} \arccos\left(\frac{L}{\sqrt{L^2 + \pi^2}}\right), \quad L = \ln\left(\frac{s}{\Lambda^2}\right) \tag{5}$$

APT couplings.

4a. Minkowskian – π^2

Summation of π^2 -terms results in 1–loop expression

$$\frac{1}{\beta_0 L} \to \bar{\alpha}_M^{(1)}(s) = \mathfrak{A}_1^{(1)}(s) = \frac{1}{\pi\,\beta_0}\arccos\frac{L}{\sqrt{L^2+\pi^2}}$$

$$\left[\mathfrak{A}_1^{(1)}(s)\right]_{L>0} = \frac{1}{\pi\beta_0}\arctan\frac{\pi}{L}\,;\ L = \ln\frac{s}{\Lambda^2}$$

Radiushkin 82

At the same time, square and cube of $\bar{\alpha}_s^{(1)}$ transform into

Krasnikov & A.Pivovarov 82 ; Bjorken '89

$$\mathfrak{A}_2^{(1)} = \frac{1}{\beta_0^2\,[L^2+\pi^2]}\,,\quad \mathfrak{A}_3^{(1)} = \frac{L}{\beta_0^3\,[L^2+\pi^2]^2}\,,$$

which are not powers of $\alpha_M^{(1)}(s)$. They are ghost-free **[Milton-Solovtsov '97**]

4. Minkowskian & Euclidean APT couplings $\bar{\alpha}_M$, $\bar{\alpha}_E$

RG defined invariant coupling $\bar{\alpha}(Q^2)$, is a real function of space-like argument Q^2 . It effectively sums up UV logs in expression with ghost trouble. At the 1-loop QCD case

$$\bar{\alpha}_s^{(1)}(Q^2) = \frac{\alpha_s}{1 - \beta_0\,\alpha_s\,\ln(Q^2/\mu^2)} = \frac{1}{\beta_0\,\ln(Q^2/\Lambda^2)}$$

Instead, in APT one has two diverse ghost-free couplings

Minkow : $$\bar{\alpha}_M^{(1)}(s) = \frac{1}{\pi\,\beta_0}\arccos\frac{L}{\sqrt{L^2+\pi^2}}\Big|_{L>0} = \frac{1}{\pi\beta_0}\arctan\frac{\pi}{L}\,;$$

Euclid : $$\alpha_E^{(1)}(Q^2) = \frac{1}{\beta_0}\left[\frac{1}{\ell} - \frac{\Lambda^2}{Q^2-\Lambda^2}\right]\,;\ \ell = \ln\frac{Q^2}{\Lambda^2}\,,\quad L = \ln\frac{s}{\Lambda^2}\,.$$

see Fig.1

4. Euclidean : Källen-Lehmann Analyticity

[APT – Sh., Solovtsov '96-98]

APT uses the Q^2 analyticity imperative. For 1-loop QCD

$$\mathbb{A}\left[\bar{\alpha}_s^{(1)}=\frac{1}{\beta_0\ell}\right]\Rightarrow\alpha_E^{(1)}(Q^2)=\frac{1}{\beta_0}\left[\frac{1}{\ell}-\frac{\Lambda^2}{Q^2-\Lambda^2}\right]\;;\;\ell=\ln\frac{Q^2}{\Lambda^2}\,.$$

For coupling $\bar{\alpha}_s^{(1)}$ squared

$$\frac{1}{\ell^2}\Rightarrow\frac{1}{\ln^2(Q^2/\Lambda^2)}+\frac{Q^2\Lambda^2}{(Q^2-\Lambda^2)^2}=\beta_0^2\,\mathcal{A}_2^{(1)}(Q^2)\neq\left(\beta_0\,\mathcal{A}_1^{(1)}\right)^2$$

By construction, APT Euclidean $\mathcal{A}_k$ and Minkowskian $\mathfrak{A}_k$ functions(for 2-loop etc. also)

★ are free of unphysical singularities;

★ at weak-coupling limit $\Rightarrow$ PT + RG; $\bar{\alpha}_s^k$

★ Include non-perturbative structures $Q^2/\Lambda^2\sim exp(-\frac{1}{\alpha_s})$.

4. Comparing $\bar{\alpha}_s$ with α_E and α_M

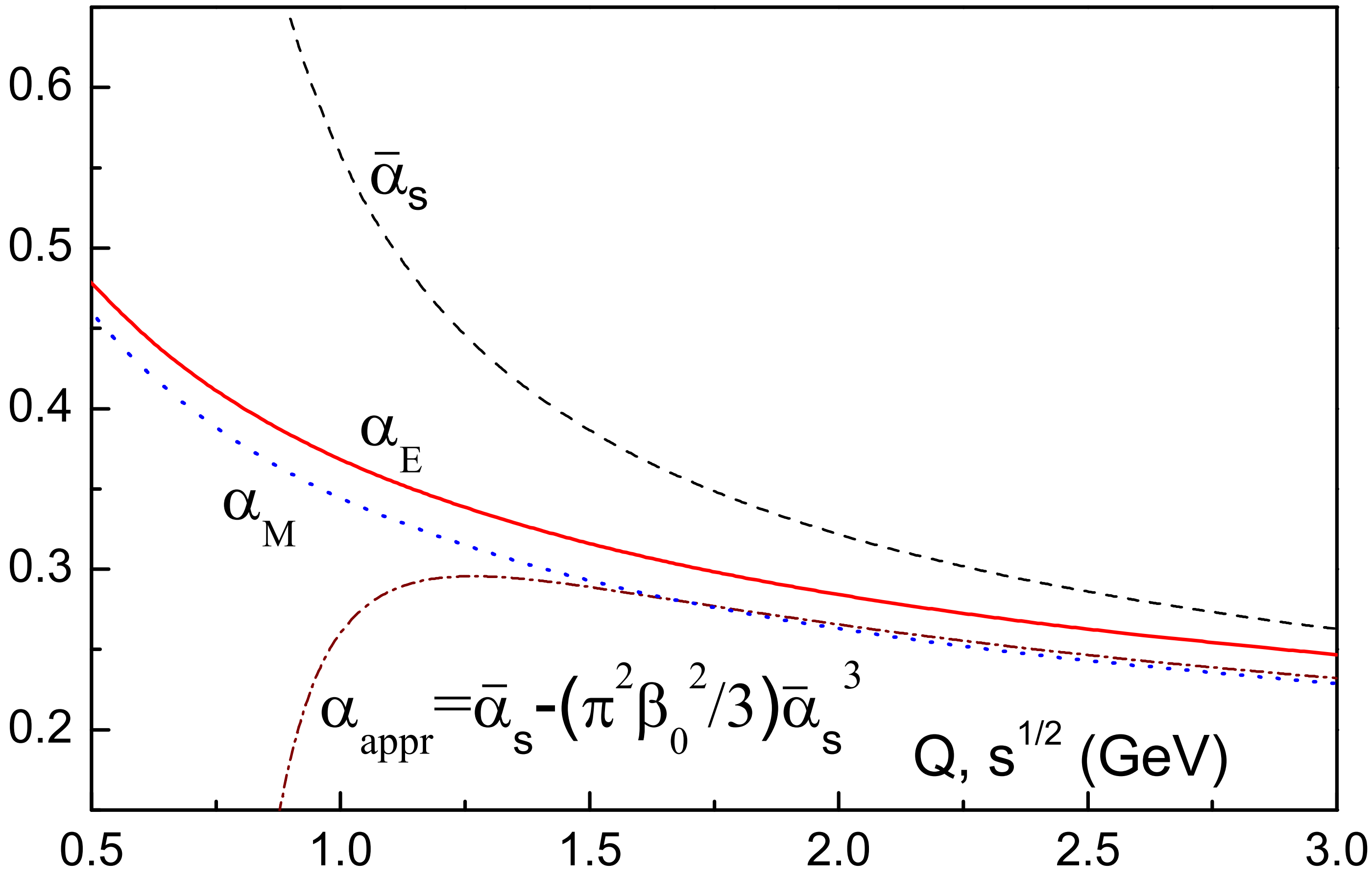

Figure 1: Comparison of usual QCD coupling $\bar{\alpha}_s$ with Euclidean α_E and Minkowskian one α_M in a few GeV region.

[a] CALC '06, Dubna

Minkowskian vs. Euclidean [Sh.- 2000/01]

Minkowskian and Euclidean ghost-free functions, are related by $\mathbb{D}$, $\mathbb{R}$ transformations: 1-loop case

$$\mathbb{D}\left[\frac{1}{\pi}\arccos\frac{L}{\sqrt{L^2+\pi^2}}\right] = \frac{1}{\ln(Q^2/\Lambda^2)} - \frac{\Lambda^2}{Q^2-\Lambda^2}\,,$$

$$\mathbb{R}\left[\frac{1}{\ln^2(Q^2/\Lambda^2)} + \frac{Q^2\Lambda^2}{(Q^2-\Lambda^2)^2}\right] = \frac{1}{[L^2+\pi^2]}\,,$$

Generally, $\mathcal{A}_k(Q^2) = \mathbb{D}\left[\mathfrak{A}_k\right]\,, \quad \mathfrak{A}_k(s) = \mathbb{R}\left[\mathcal{A}_k\right]\,.$

Accordingly,

$$\mathbb{D}\left[R(s) = \sum_k r_k\,\mathfrak{A}_k(s)\right] \Rightarrow D(Q^2) = \sum_k d_k\,\mathcal{A}_k(Q^2)\,.$$

[a] CALC '06, Dubna

4. Main Results of APT. One-loop case. Cont'd

Higher functions $\mathcal{A}_k$, $\mathfrak{A}_k$ can be defined via differential recursive relations

$$\frac{1}{k}\frac{d\mathcal{A}_k^{(1)}}{dL} = -\beta_0\,\mathcal{A}_{k+1}^{(1)} + o(\beta_1), \qquad \frac{1}{k}\frac{d\mathfrak{A}_k^{(1)}}{dL} = -\beta_0\,\mathfrak{A}_{k+1}^{(1)} + \ldots$$

The 2nd and 3rd one-loop functions are

$$\mathcal{A}_2^{(1)}(l) = \frac{1}{\beta_0^2}\left(\frac{1}{l^2} - \frac{e^l}{(e^l-1)^2}\right), \quad \mathfrak{A}_2^{(1)}(L) = \frac{1}{\beta_0^2}\,\frac{1}{L^2+\pi^2}\,;$$

$$\mathcal{A}_3(l) = \frac{1}{\beta_0^3}\left(\frac{1}{l^3} - \frac{1}{2}\,\frac{e^l+e^{2l}}{(e^l-1)^3}\right), \quad \mathfrak{A}_3(L) = \frac{1}{\beta_0^3}\frac{L}{(L^2+\pi^2)^2}\,.$$

[a] CALC '06, Dubna

4c. Nonpower ghost-free sets $\{\mathcal{A}_k\}$, $\{\mathfrak{A}_k\}$

qualitatively, are similar:

I. First ones, new couplings, α_E, α_M,

♢ are monotonic and IR finite $= 1/\beta_0 \simeq 1.4$

♢ in the UV limit $\sim 1/\ln x \sim \bar{\alpha}_s(x)$.

II. All the other functions $(k \geq 2)$:

♡ start from zero $\mathcal{A}_k(0), \mathfrak{A}_k(0)) = 0;$

♡ in the UV limit $\sim 1/(\ln x)^k \sim \bar{\alpha}_s^k(x)$.

♡ 2nd, $\mathcal{A}_2$, $\mathfrak{A}_2$ obey max at $\sim \Lambda^2$.

♡ Higher ones $\mathcal{A}_{k\geq 3}$; $\mathfrak{A}_{k\geq 3}$ oscillate near Λ^2 with $k-1$ zeroes $\to$ RS-insensitivity and to better convergence at low-energy region. – see Figs 3.

4b. $E \to M$ "distorting mirror" [Milton-Solovtsov '98]

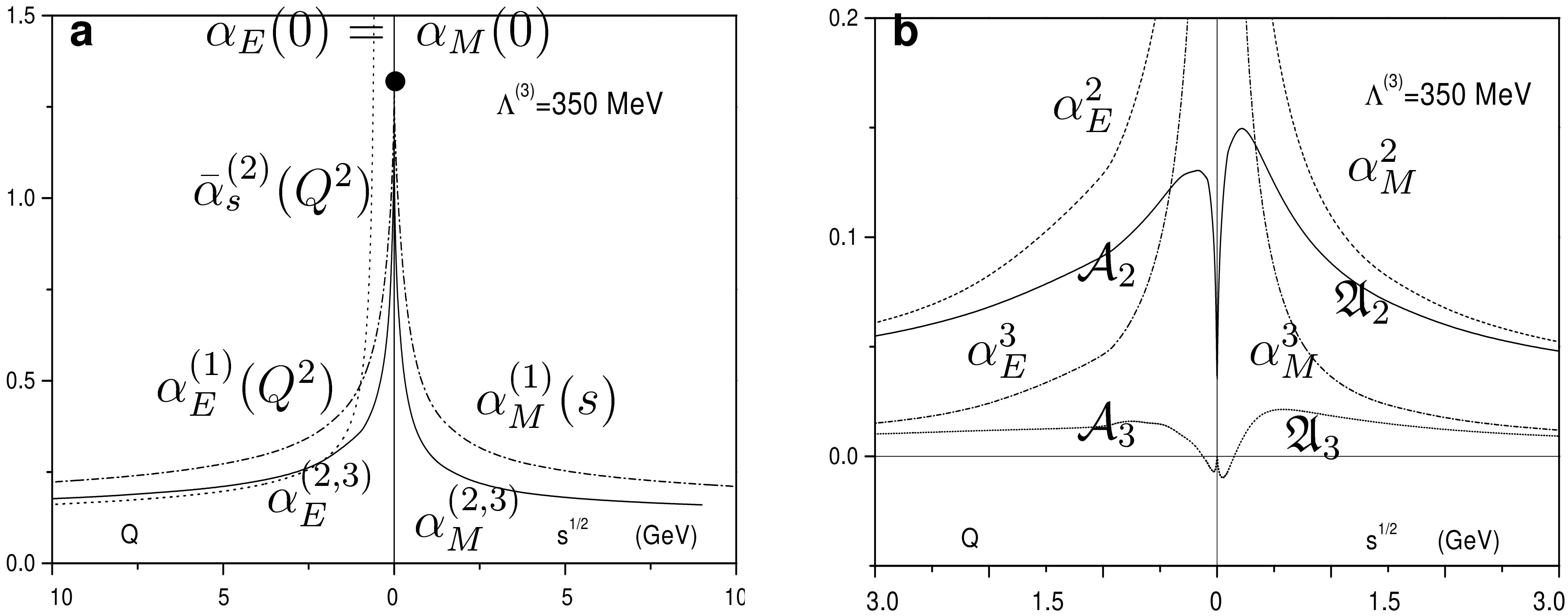

Figure 3: **a** – *Space-like and time-like analytic couplings in few* GeV *domain.* **b** – *"Distorted mirror symmetry" for global expansion functions. All the curves in* **1b** *correspond to exact two–loop solutions expressed in terms of Lambert function.*

Non-power expansions for QCD observables

These effective couplings are related by integral operation

$$\alpha_E(Q^2) = Q^2 \int_0^\infty \frac{\bar{\alpha}_M(s)\, ds}{(s+Q^2)^2} \equiv \mathbb{D}\left[\bar{\alpha}_M\right](Q^2) \qquad (\mathbb{D})$$

treated as **integral transformation $\mathbb{D}$.**
The same transformation relates

$$\mathfrak{A}_k(s) \to \mathcal{A}_k(Q^2) \equiv \mathbb{D}\left[\mathfrak{A}_k\right](Q^2)$$

expansion functions for observables

$$d(Q^2) = d_1 \alpha_E(Q^2) + d_2\, \mathcal{A}_2(Q^2) + d_3\, \mathcal{A}_3(Q^2) + \ldots$$

$$r(s) \;=\; d_1\, \alpha_M(s) \;+\; d_2\, \mathfrak{A}_2(s) \;+\; d_3\, \mathfrak{A}_3(s) \;+\; \ldots$$

4b. $\int$ Transformations: $\bar{\alpha}_s(Q^2) \to \bar{\alpha}_E(Q^2) \Leftrightarrow \bar{\alpha}_M(s)$

Contour integration [by **Radiyshkin-Krasnikov-Pivovarov '82**]

$$\bar{\alpha}_s(Q^2) \to \frac{i}{2\,\pi}\int_{s-i\varepsilon}^{s+i\varepsilon}\frac{dz}{z}\,\bar{\alpha}_s(-z) = \bar{\alpha}_M(s) \equiv \mathbb{R}\left[\bar{\alpha}_s\right](s)$$

results in free of ghost expression with π^2 terms summed.
Reverse transformation [**Solovtsov + D.Sh. '96-'98**] $\mathbb{D} = [\mathbb{R}]^{-1}$
yields ghost-free expression with subtracted singularity,
e.g., 1-loop case:

$$\bar{\alpha}_s^{(1)}(Q^2) = \frac{1}{\beta_0\,\ell} \to \mathbb{R}\left[\bar{\alpha}_s\right] = \bar{\alpha}_M^{(1)}(s) = \frac{1}{\pi}\arctan\left(\frac{\pi}{L}\right) \to$$

$$\to \mathbb{D}[\bar{\alpha}_M^{(1)}] = \frac{1}{\beta_0}\left[\frac{1}{\ell} + \frac{\Lambda^2}{\Lambda^2 - Q^2}\right] \equiv \bar{\alpha}_E^{(1)}(Q^2)\,.$$

Logic of the scheme is displayed at Fig.2

Sketch of the global APT algorithm

The most convenient form of APT formalism uses a *spectral density* $\rho(\sigma)$ taken from perturbative input. Then all involved functions in the 3 above mentioned pictures look like

$$\mathcal{A}_k = \frac{1}{\pi}\int\limits_0^\infty \frac{\rho_k(\sigma)\,d\sigma}{\sigma + Q^2}\,, \qquad \mathfrak{A}_k = \frac{1}{\pi}\int\limits_s^\infty \frac{d\sigma}{\sigma}\,\rho_k(\sigma)\,,$$

At the 1-loop case

$$\rho_1^{(1)} = \frac{1}{\beta_0\,[L_\sigma^2 + \pi^2]}\,; \qquad L_\sigma = \ln\frac{\sigma}{\Lambda^2}\,; \quad k\,\beta_0\,\rho_{k+1}^{(1)}(\sigma) = -\frac{d\,\rho_k^{(1)}(\sigma)}{d\,L_\sigma}$$

These expressions were generalized for higher-loops case and for real QCD with transitions across quark thresholds. They were successively used for fitting of various data.

gAPT – closed scheme

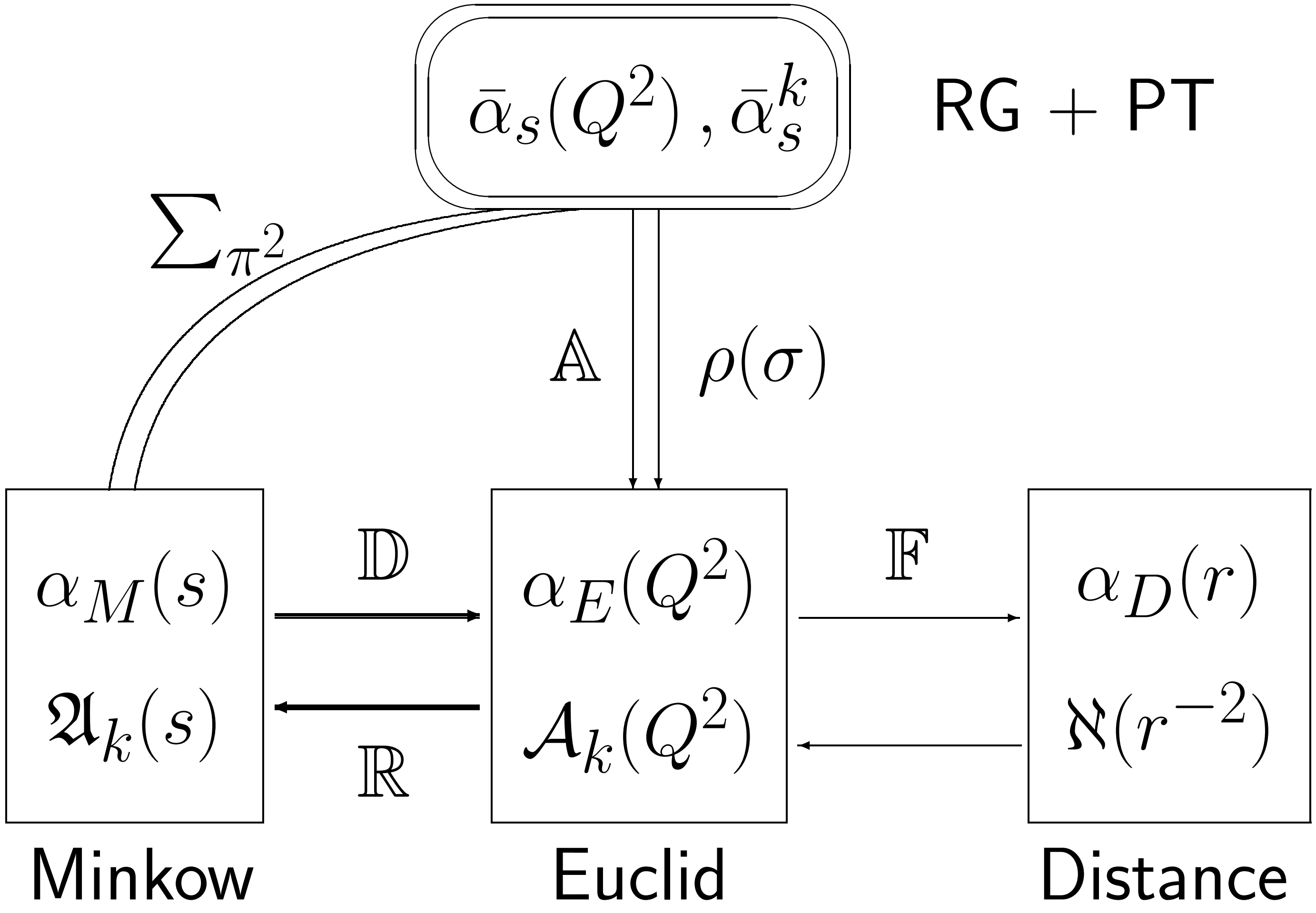

Fig.2 *Logic of APT scheme.*

APT quick – loop convergence

Instead of “PDG–canonized" power-in-$\bar{\alpha}_s$, like

$$d_\pi(Q) = d_1 \bar{\alpha}_s(Q) + d_2\, \bar{\alpha}_s^2 + \mathbf{d_3}\, \bar{\alpha}_\mathbf{s}^\mathbf{3}\ ;$$

one use non-power expansions

$$d_{\rm an}(Q^2) = d_1\, \alpha_E(Q^2) + d_2\, \mathcal{A}_2(Q^2) + \mathbf{d_3}\, \mathcal{A}_\mathbf{3}(\mathbf{Q^2}) + \ldots$$

$$r_\pi(s) = d_1 \alpha_M(s) + d_2\, \mathfrak{A}_2(s) + \mathbf{d_3}\, \mathfrak{A}_\mathbf{3}(\mathbf{s}) + \ldots$$

Contributions in %% of 1, 2 and 3-loop terms into observables

Process: E/M		*PT*			*APT*		
GLS Sum R.	E	65	24	**11**	75	21	**4**
Bjorken SR	E	55	26	**19**	80	19	**1**
Incl τ-decay	M	55	29	**16**	88	11	**1**
$e^+e^- \to$hadr	M	96	8	-4	92	7	**.5**
$Z_o \to$ hadr.	M	98.6	3.7	-2.3	96.9	3.5	-**.4**

[a] CALC ’06, Dubna

The APT Minkowskian $n_f = 5$ fit

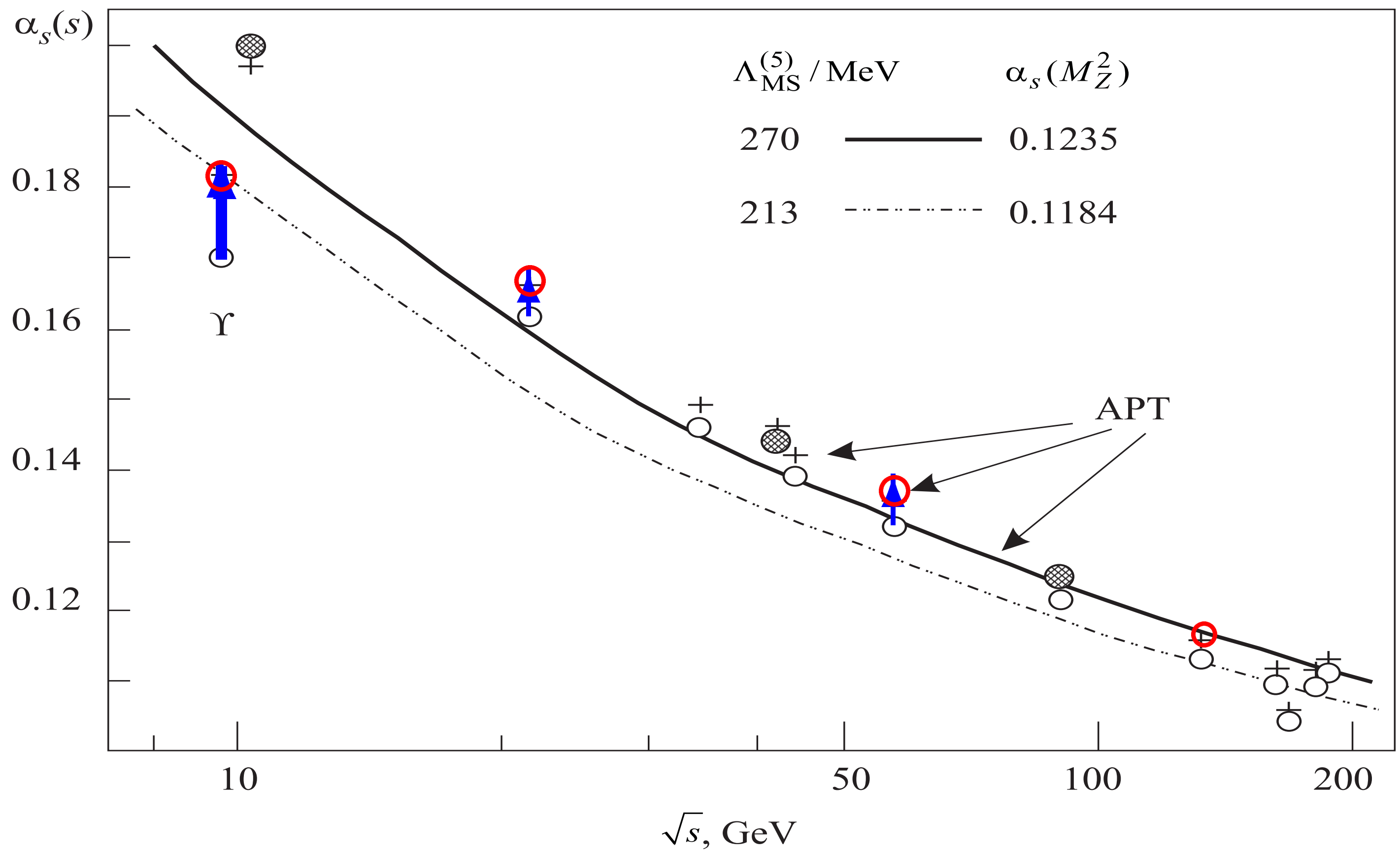

Figure 4: By circles and dot-and-dash curve, we give usual $\bar{\alpha}_s$ $(\Lambda^{(5)} = 213\, MeV\,;\ \alpha_s(M_Z) = 0.118)$ taken from Bethke review in $f = 5$ time–like region. APT crosses (+), red circles and solid curve account for π^2 corrections. It results in $\Lambda^{(5)}_{\overline{\rm MS}} = 270\, MeV\,;\ \alpha_s(M_Z) = 0.124\,.$

Simple Analytic Models for 3-loop $\mathfrak{A}_k$ and $\mathcal{A}_k$

2-,3-loop solutions for Minkowskian $\mathfrak{A}$ and Euclid. couplings involve special Lambert function W_{-1}; being cumbersome. Instead, for the Low-Energy region, we devised a class of

2-parameter "1-loop-like" analytic M o d e l s :

$$\mathfrak{A}_1^{mod}(s) = \frac{1}{\pi\,\beta_0}\,\arctan\frac{\pi}{L_2}\,, \qquad \mathfrak{A}_2^{mod} = \frac{1}{\beta_0^2\,[L_2^2+\pi^2]};$$

$$\alpha_E^{mod}(Q^2) = \frac{1}{\beta_0}\left[\frac{1}{\ell_2} - \frac{1}{e^{\ell_2}-1}\right];\; \beta_0^2\,\mathcal{A}_2^{mod}(Q^2) = \frac{1}{\ell^2} + \frac{e^{\ell_2}}{(e^{\ell_2}-1)^2},$$

$$\ell_2 = \ell + b\,\ln\sqrt{(\ell+\Delta)^2+\kappa\pi^2}\,; \qquad L_2 = L + b\ln\sqrt{(L+\Delta)^2+\kappa\pi^2}$$

The 1-parametric variant $(\Delta = 0\,,\ \kappa = 2)$ gives 3-5 %% error **[Zayakin+D.Sh. this Conf.]** for $1\ \mathrm{GeV} < \sqrt{s},\ |Q|$; its 2-parametric version $(\Delta = 1\,,\ \kappa = 2)$ **[Pasechnik+D.Sh. '06]** is of 1 per cent level.

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The scale issue

Accuracy of experiment in hadron physics is rising quickly. Theory Reaction is to involve higher loops in analysis. However, in many cases 3- and 4-loops effects are small.

The scale uncertainty. Besides “direct" data error that, e.g., in Upsilon 1S decay, is 2% for R_Υ , there is a “hidden" issue in theory due to choice of argument $L = \ln(s_\Upsilon/\Lambda^2)$.

This reference scale issue is actual for QCD analysis of all low energy data. The scale effect enlarges quickly with the uncertainty rise.

E.g., in Upsilon 1S decay, for its value 1 GeV, i.e., for 8–9 GeV interval, we have $205 \leq \Lambda_5 \leq 240\,\mathrm{MeV}$, while to the 7–9 GeV case — $180 \leq \Lambda_5 \leq 240\,\mathrm{MeV}$.

The scale issue, II

Scale issue is intrinsic RG problem:
Fisrt, the common, *"vulgaris"*, version of RG algorithm corresponds to the UV (massless) case with simplified definition of QCD coupling $\bar{\alpha}(q^2/\mu^2)$. Here, it is assumed that vertex function $\Gamma(q_1^2, q_2^2, q_3^2)$ entering $\bar{\alpha}$ is taken with equal arguments.

Second, condition $q^2 \gg m^2$ is used. Both are not valid for low energy QCD.

On these items, we address to mass-dependent RG formalism **[D.Sh-92; Dokshitzer & D.Sh.-95]**

Meanwhile, scale uncertainty essentially reduces the value of theoretical analysis of Υ decay. In the case, the scale error dominates over experimental and Model ones. This issue is worth urgent further examining.

Conclusion

1. In PT QCD use:
   - **Denominator eq.** instead of PDG (9.5);
   - ghost-free APT $\alpha_E(Q)$, $\alpha_M(s)$ at low energy.
2. The APT implies that expansions – in powers of singular $\bar{\alpha}_s(Q^2$ or $s)$ – should be changed for **Non-power PT expansions**, e.g.,

$$r_{APT}(s) = r_1\alpha_M(s) + r_2\,\mathfrak{A}_2(s) + r_3\,\mathfrak{A}_3 \, + \ldots \, ,$$

   in sets of functions $\{A_k(Q^2)\}$ and $\{\mathfrak{A}_k(s)\}$ with **much better convergence**.

[a] CALC '06, Dubna

Conclusion; II

3. Hence, in the both **H**adronization and **IR** regions **higher-loop-terms do not determine the level of accuracy**.

4. The **bottleneck now is the scale uncertainty** that essentially reduces the value of theoretical analysis. In the $\Upsilon\, 1S$ decay and some other cases with high data resolution, the scale error dominates over data ones. The **scale issue is worth urgent further examining**.

[a] CALC '06, Dubna