Gravity in two dimensions

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- Dilaton gravity from:
  – dimensional reduction of gravity;
  – strings.

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[D.Grumiller, W.Kummer and D.V., Phys. Repts. 369 (2002) 327-430]

The Einstein-Hilbert action in n-dimensions:

$$S_{EH} \sim \int d^n x \sqrt{-g} R(g)$$

(R is the scalar curvature). Degrees of freedom:

– metric g – symmetric $n \times n$ matrix: $\frac{n(n+1)}{2}$

– diffeomorphisms – complex vector ghosts: $-2n$

– total: $\frac{n(n-3)}{2}$

4 dimensions: 2 degrees of freedom (two gravitons)

2 dimensions: -1 degree of freedom! The EH action in 2D is proportional to the Euler characteristic of the manifold.

One has to add at least one degree of freedom.

2D dilaton gravity is an interacting system of gravity and a scalar field X (called dilaton) in two dimensions.

$$S = \int_M d^2x\sqrt{-g}\left[\frac{R}{2}X - \frac{U(X)}{2}(\nabla X)^2 + V(X)\right]$$

NO propagating degrees of freedom! More general action:

$$S = \int_M d^2x\sqrt{-g}\left[\frac{R}{2}X + \mathcal{V}(-(\nabla X)^2, X)\right]$$

First-order form:

$$S = \int_M [X_a De^a + X d\omega + \epsilon\mathcal{V}(X_a X^a, X)]$$

where e^a is a moving frame (zweibein), X^a are auxiliary fields, ω is a spin connection, ϵ is a volume two-form.

$$e^a_\mu e^a_\nu = g_{\mu\nu}$$

First and second order actions are classical and quantum equivalent.

The simplest model: **Jackiw-Teitelboim**.

$$S = \int_M d^2x \sqrt{-g} X (R - \Lambda),$$

where Λ is the cosmological constant. Variation w.r.t. X gives a 2D version of the Einstein equation

$$R = \Lambda.$$

Note, that in 2D the Riemann tensor is totally defined by the scalar curvature:

$$R_{\mu\nu\rho\sigma} \sim \varepsilon_{\mu\nu} \varepsilon_{\rho\sigma} R$$

This model was introduced in 1983 by Jackiw and Teitelboim, but the equation of motion (constant curvature) was first considered by Barbashov, Nesterenko & Chervyakov in 1979.

Spherical reduction

Consider a spherically symmetric line element in D-dimensions:

$$(ds)^2_D = g_{\mu\nu}(x)dx^\mu dx^\nu - X^{\frac{2}{D-2}}(x)(d\Omega)^2_{S^{D-2}}$$

$\mu, \nu = 0, 1$. Substitution in the Einstein-Hilbert action

$$S_D = \int d^D x \sqrt{-g_D} R_D$$

yields:

$$S \sim \int_M d^2 x \sqrt{-g} \left[XR + \frac{(D-3)}{(D-2)} \frac{(\nabla X)^2}{X} - (D-2)(D-3) X^{\frac{D-4}{D-2}} \right]$$

This is a dilaton gravity action.
NB: Schwarzschild black hole is here!

Strings

Non-linear sigma model action for closed bosonic string:

$$S^{[\sigma]} = \frac{1}{4\pi\alpha'}\int d^2\xi\sqrt{-h}\left[h^{ij}g_{\mu\nu}\partial_i X^\mu \partial_j X^\nu + \alpha'\Phi\mathcal{R}\right]$$

α' is inverse string tension; ξ^i, $h_{ij}(\xi)$, $\mathcal{R}$ are coordinates, the metric tensor and the scalar curvature on the world surface of the string. $X^\mu(x)$ are coordinates of the string in the target manifold. $G_{\mu\nu}$ (metric on the target manifold) and $\Phi(X)$ (dilaton, but a different one) play the role of (infinitely many!) couplings.

Conformal "anomaly":

$$2\pi T_i^i = \beta^\Phi \mathcal{R} + \beta^g_{\mu\nu} h^{ij}\partial_i X^\mu \partial_j X^\nu$$

where β^Φ and $\beta^g_{\mu\nu}$ are beta functions.

Conformal invariance of strings:

$$\beta^\Phi = \beta^g_{\mu\nu} = 0$$

Leading order in α':

$$\frac{\beta^\Phi}{\alpha'} = -\frac{\lambda^2}{4\pi^2} - \frac{1}{16\pi^2}\left(2(\nabla\Phi)^2 - 4\nabla^2\Phi - R\right),$$
$$\beta^g_{\mu\nu} = R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\Phi$$

$R_{\mu\nu}$ and R are the Ricci tensor and the scalar curvature on the *target* space.

Conformal invariance of strings to the leading order is equivalent to the equations of motion following from the (classical) action on the target space:

$$S = \int d^D X \sqrt{-g} e^{-2\Phi}\left[R + 4(\nabla\Phi)^2 - 4\lambda^2\right]$$

After obvious identifications $X = e^{-2\Phi}$ this is a dilaton gravity action again.

Poisson sigma models

Let X^I be target space coordinates (not the string ones), A_I be one-forms (gauge fields). The action:

$$S^{\mathsf{PSM}} = \int \left[dX^I \wedge A_I + \frac{1}{2}\mathcal{P}^{IJ} A_J \wedge A_I \right]$$

$\mathcal{P}^{IJ}(X)$ is called the Poisson tensor.

Dilaton gravity:

$$X^I \to (X, X^a), \qquad A_I \to (\omega, e^a)$$
$$\mathcal{P}^{aX} = X^b \epsilon_b{}^a, \qquad \mathcal{P}^{ab} = \mathcal{V}\epsilon^{ab}$$

Symmetries:

$$\delta X^I = \mathcal{P}^{IJ}\alpha_J$$
$$\delta A_I = d\alpha_I - \left(\partial \mathcal{P}^{JK}/\partial X^I\right) A_J \alpha_K$$

(in the case of dilaton gravity these are Lorentz and diffeomorphism trafos)

Schouten-Nijenhuis bracket:

$$\{X^I, X^J\} = \mathcal{P}^{IJ}$$

(must satisfy usual Jacobi identity).

This is "non-commutative geometry". The same structure appears in open strings and Dirichlet branes (where $\mathcal{P} \sim B_{\mu\nu}$).

Equations of motion:

$$\begin{aligned} &dX^I + \mathcal{P}^{IJ} A_J = 0 \\ &dA_I + \frac{1}{2}\left(\partial \mathcal{P}^{JK} / \partial X^I\right) A_K \wedge A_J = 0 \end{aligned}$$

Let $C(X)$ be such that $\{X^I, C(X)\} = 0$. Then

$$dC = -A_J \{X^J, C\} = 0$$

Infinite number of conservation laws!

Dilaton gravity: $\mathcal{P}^{IJ}$ is an antisymmetric 3×3 matrix. There is always 1 zero eigenvalue. $C(X)$ always exists. All dilaton gravities are exactly soluble.

One can also prove:

• In a suitable coordinate system all solutions depend on one coordinate only (Birkhoff theorem).

• Generic solutions are "black holes", i.e. they have singularities and horizons. The constant C plays the role of the black hole mass.

Schwarzschild gauge:

$$(ds)^2 = \xi(r)dt^2 - \xi(r)^{-1}dr^2$$

$\xi(r) = 0$ - horizon, $\xi(r) = \infty$ - curvature singularity.

Spherically reduced gravity ($D = 4$):

$$\xi(r) = 1 - \frac{2M}{r}$$

String gravity:

$$\xi(r) = 1 - \frac{M}{\lambda}e^{-2\lambda r}$$

In both cases, $r \to \infty$ is flat Minkowski space.
$M \sim C$.

Hawking radiation. This is a quantum effect of radiation of particles from the black hole. We need quantum $T_{\mu\nu}$ at the asymptotic region (far away from the black hole).

Consider (a) a fixed black hole; (b) a minimally coupled scalar field:

$$S = \frac{1}{2}\int d^2x\sqrt{-g}(\nabla\phi)^2$$

This action is conformally invariant. Classically $T^\mu_\mu = 0$. Quantum effects lead to the conformal anomaly:

$$T^\mu_\mu = \frac{1}{24\pi}R$$

Consider the conformal gauge and the light-cone coordinates:

$$(ds)^2 = 2e^{2\rho}dx^+dx^-\, ,\text{ë} \qquad \rho = \frac{1}{2}\ln\xi.$$

Then

$$T^\mu_\mu = 2e^{-2\rho}T_{+-}$$

Conservation law (follows from the diffeomorphism invariance):

$$\nabla_\mu T^\mu_\nu = 0$$

In the light cone gauge on static backgrounds ($\partial_z = \xi(r)\partial_r$):

$$(\partial_z - 2(\partial_z \rho))T_{+-} = \partial_z T_{--}$$

This is an ODE which defines T_{--} (radiation!) up to an integration constant:

$$T_{--} = \frac{1}{24\pi}(\partial_z^2 \rho - (\partial_z \rho)^2) + t$$

t is fixed by the Unruh boundary condition: $T_{--}(\text{horizon}) = 0$. Stefan-Boltzmann law:

$$T_{--}|_{asympt} = \frac{\pi}{6} T_H^2$$

Spherically symmetric black hole:

$$T_H \sim 1/M$$

The specific heat is negative. Black holes evaporate leading to the information paradox.

String gravity:

$$T_H = \frac{\lambda}{2\pi}$$

Loop corrections (may be calculated in 2D only):

$$T_H = \frac{\lambda}{2\pi}\left(1 - \frac{\lambda}{48\pi M}\right)$$

The specific heat is positive.

Preliminary conclusions

1. Dilaton gravity models appear in a number of physical applications (because they are the most general covariant models in 2D).

2. Without matter fields they can be solved exactly.

3. Quantum dilaton gravities provide us with an interesting insight into the life of quantum black holes.

Noncommutative gravity in two dimensions. Noncommutativity is a way to give a precise mathematical meaning to the idea that the space-time structure is modified at short distances. (Appear in: deformation quantization; strings; quantum Hall effect).

NC QFT: Douglas M. R. and Nekrasov N. A., Noncommutative field theory, Rev. Mod. Phys. **73** (2001) 977-1029 [arXiv:hep-th/0106048].

Classical NC gravity in 2D: S. Cacciatori, A. H. Chamseddine, D. Klemm, L. Martucci, W. A. Sabra and D. Zanon, Noncommutative gravity in two dimensions, Class. Quant. Grav. **19** (2002) 4029 [arXiv:hep-th/0203038].

Quantum NC gravity in 2D: DV, hep-th/0406163

Gelfand-Neimark Th.: Any commutative associative ($C*$) algebra is an algebra of functions on a manifold.

Take an *noncommutative* associative algebra. Then it defines a *noncommutative* manifold. We like to have:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}$$

with, for simplicity, a constant θ. Then the corresponding associative but noncommutative product (called the Moyal product) reads:

$$f \star g = f(x) \exp\left(\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu\right) g(x)$$

From now on we have to use this product everywhere. For example, the field strength for a connection A_μ is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + A_\mu \star A_\nu - A_\nu \star A_\mu$$

Note: non-linearities even in the abelian case! Not all usual gauge groups can be closed.

2 dimensions: it is possible to construct a gravity theory starting with a topological $U(1,1)$ model:

$$S = \int \mathrm{tr}\,(\Phi \star F)$$

Gauge transformations:

$$A \to g_\star^{-1} \star A \star g_\star + g_\star^{-1} \star d g_\star, \qquad \Phi \to g_\star^{-1} \star \Phi \star g_\star$$

F and Φ carry a four-dim gauge index. Let:

$$\Phi^i = (l\phi^a, \phi, \psi), \qquad A^i_\mu = (e^a_\mu l^{-1}, \omega_\mu, b_\mu)\,,$$

where $a = 0, 1$, and the scale l is related to the cosmological constant $\Lambda = -1/l^2$. Then:

$$S = \frac{1}{4}\int d^2x\, \varepsilon^{\mu\nu} \left[\phi_{ab} \star \left(R^{ab}_{\mu\nu} - 2\Lambda e^a_\mu \star e^b_\nu\right) - 2\phi_a \star T^a_{\mu\nu}\right]$$

where $\phi_{ab} := \phi\varepsilon_{ab} - i\eta_{ab}\psi$. η_{ab} is the "flat" metric, $\eta_{ab} = \mathrm{diag}(+1, -1)$.

NC curvature:

$$R^{ab}_{\mu\nu} = \varepsilon^{ab}\left(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu + \frac{i}{2}[\omega_\mu, b_\nu] + \frac{i}{2}[b_\mu, \omega_\nu]\right)$$
$$+\eta^{ab}\left(i\partial_\mu b_\nu - i\partial_\nu b_\mu + \frac{1}{2}[\omega_\mu, \omega_\nu] - \frac{1}{2}[b_\mu, b_\nu]\right)$$

NC torsion:

$$T^a_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu + \frac{1}{2}\varepsilon^a{}_b\left(\{\omega_\mu, e^b_\nu\} - \{\omega_\nu, e^b_\mu\}\right)$$
$$+\frac{i}{2}\left([b_\mu, e^a_\nu] - [b_\nu, e^a_\mu]\right) .$$

Gauge symmetries: 2 diffeomorphisms, 1 Lorentz boost, 1 additional $U(1)$-symmetry (with the gauge field b_μ). Important: there is indeed Lorentz and diffeomorphism invariance although $\theta^{\mu\nu}$ is fixed to be a constant!

In the commutative limit ($\theta \to 0$) the pair (b_μ, ψ) decouples. The rest of the fields describes the Jackiw-Teitelboim model (e.o.m first introduced by Barbashov, Nesterenko, Chervyakov).

Classical equations of motion

$$F_{\mu\nu} = 0,$$
$$D\Phi = d\Phi + [A, \Phi] = 0$$

can be solved immediately:

$$A_\mu = g_\star^{-1} \star \partial_\mu g_\star$$
$$\Phi = g_\star^{-1} \star (constant) g_\star$$

(Although in the second order formulation and in less convenient variables this is not so obvious)

Path integral **Quantization** of NC gravity in 2D. First, we have to fix a gauge. Let us choose the temporal gauge:

$$e_0^+ = 0, \quad e_0^- = 1, \quad \omega_0 = 0, \quad b_0 = 0.$$

Let us introduce the notations:

$$\begin{aligned} q^i &= (e_1^+, e_1^-, \omega_1, b_1)\,, \\ p^i &= (\phi_+, \phi_-, \phi, \psi)\,, \\ \bar{q}^i &= (e_0^+, e_0^-, \omega_0, b_0)\,, \end{aligned}$$

so that the gauge conditions read: $\bar{q}^i - a^i = 0$, where $a^i := (0, 1, 0, 0)$.

Consider a set of matter fields $\{f^\alpha\}$ described by a classical action $S_{\mathrm{m}}(q, \bar{q}; f^\alpha)$. We only suppose that this action does not depend on p^i.

Let us consider the generating functional for Green functions (in the path integral representation):

$$Z(j,J) = \int \mathcal{D}q\,\mathcal{D}\bar{q}\,\mathcal{D}p\,\mathcal{D}f^\alpha \prod \delta(\bar{q}^i - a^i)\,\mathcal{F}$$
$$\times \exp\left(iS + iS_{\mathsf{m}}(q,\bar{q}; f^\alpha) + i\int d^2x (j_k q^k + J_k p^k)\right),$$

where $\mathcal{F}$ is the Faddeev-Popov determinant. In our gauge

$$\mathcal{F} = \det\left(\delta\bar{q}^i/\delta\lambda_j\right) = \det(\partial_0)^4$$

it does not depend on the fields (a derivation based on the BRST methods would be better, but since our system is of the Yang-Mills type, this one is actually enough).

J, j are external sources. We can also introduce external sources (or mean fields) for the matter, but it would change very little in the subsequent calculations. Let us define (formally) an effective action of the matter fields:

$$W_{\mathsf{m}}(q,\bar{q}) = \frac{1}{i}\ln\int \mathcal{D}f^\alpha e^{iS_{\mathsf{m}}(q,\bar{q};f^\alpha)},$$

so that we perform the integration over f and $\bar{q}$ to obtain:

$$Z(j,J) = \int \mathcal{D}q\,\mathcal{D}p \prod \delta(\vec{q}^i - a^i)\,\mathcal{F}$$
$$\times \exp\left(iS_{\text{g.f.}} + iW_{\text{m}}(q,a) + i\int d^2x(j_k q^k + J_k p^k)\right)$$

where the gauge fixed gravitational action

$$S_{\text{g.f.}} = \int d^2x\,[\phi\partial_0\omega_1 - \psi\partial_0 b_1$$
$$+\Lambda\phi e_1^+ + \phi_a\partial_0 e_1^a + \phi_-\omega_1\Big]$$

is quadratic in all fields even in the noncommutative case! This means that we have p^i everywhere only linearly. Integration over p^i produces 4 delta-functions:

$$\begin{aligned}
&\delta(\partial_0 e_1^+ + J^+)\,,\\
&\delta(\partial_0 e_1^- + \omega_1 + J^-)\,,\\
&\delta(\partial_0\omega_1 + \Lambda e_1^+ + J^3)\,,\\
&\delta(\partial_0 b_1 - J^4)\,.
\end{aligned}$$

Because of these delta-functions the fields q^i are fixed to their “classical” values:

$$\begin{aligned}
&e_1^+ = E_1^+ := -\partial_0^{-1}J^+\,,\\
&\omega_1 = \Omega_1 := -\partial_0^{-1}J^3 + \partial_0^{-2}J^+\,,\\
&e_1^- = E_1^- := -\partial_0^{-1}J^- + \partial_0^{-2}J^3 - \partial_0^{-3}J^+\,,\\
&b_1 = B_1 := \partial_0^{-1}J^4\,.
\end{aligned}$$

Integration over q results in the replacement:

$$q \to Q := (E_1^+, E_1^-, \Omega_1, B_1)$$

Finally, we obtain:

$$Z(j,J) = \exp\left(iW_{\mathsf{m}}(Q) + i\int d^2x\, Q(J)j\right) .$$

Here $W_{\mathsf{m}}(Q) := W_{\mathsf{m}}(Q, a^j)$. This equation gives a full non-perturbative result for the generating functional of the Green functions. All loop effects due to the gravity sector has been taken into account explicitly, while the integration over the matter fields has been performed symbolically (this is the best one can do without specifying the action S_{m}).

To get a better idea of what is going on let us calculate the action (by performing the Legendre transform).

$$W(P,Q) = \frac{1}{i} \ln\, Z - \int d^2x (PJ + Qj)$$

where the mean fields are defined as:

$$Q = \frac{1}{i}\frac{\delta \ln Z}{\delta j}\,, \qquad P = \frac{1}{i}\frac{\delta \ln Z}{\delta J}\,.$$

After some algebra we obtain:

$$W(P,Q) = S_{\mathrm{g.f.}}(P,Q) + W_{\mathrm{m}}(Q)\,.$$

This result means that all loop corrections due to the gravity fields disappear!

We have not discussed asymptotic conditions, boundary terms, and other “global” issues. Therefore, we call this kind of statements “local quantum triviality” although locality looses its meaning in noncommutative theories.

Note: there ARE global quantum effects.

How can one calculate **the matter effective action**?

"Normal way": find a suitable classical action which respects all essential symmetries and then quantize. This does not work here since the symmetries are realized in a very non-trivial way and because of the combinatorial complexity. To reduce this complexity we choose

$$b_\mu = 0$$

and fix the conformal gauge

$$e_\mu^a = e^\rho \delta_\mu^a.$$

Now we can solve the torsion equation $T^a_{\mu\nu} = 0$ which reads

$$2e^{-\rho} \star (\partial_\mu e^\rho) = -e^{-\rho} \star \widehat{\omega}_\mu \star e^\rho - \widehat{\omega}_\mu \,,$$

where

$$\widehat{\omega}_\mu = \varepsilon_\mu{}^\nu \omega_\nu = \eta_{\mu\rho} \varepsilon^{\rho\nu} \omega_\nu \,.$$

Then

$$\widehat{\omega}_\mu = \sum_{n=1}^{\infty} c_n [\ldots [[\partial_\mu \rho, \rho], \rho], \ldots],$$

all $c_{2k} = 0$, and

$$\widehat{\omega}_\mu = -\partial_\mu \rho + \frac{1}{12}[[\partial_\mu \rho, \rho], \rho] + \mathcal{O}(\rho^5).$$

One can define the scalar curvature density as

$$\mathcal{R} = \frac{1}{2}\varepsilon^{\mu\nu}\varepsilon_{ab}R^{ab}_{\mu\nu} = 2\partial_\mu \widehat{\omega}_\mu \, .$$

Note, that this is still a total derivative. Thus it defines a topological charge. (All contractions are performed with the flat metric).

We have defined all main geometrical quantities.

In the one-loop approximation (or for quadratic matter actions):

$$W_{\mathrm{m}} \sim \ln \det D$$

with a suitable operator D. We are going to look for a deformation of the operator algebra (instead of analysing classical action).

Scalar product:

$$\langle \kappa_1, \kappa_2 \rangle = \int d^2x \, \kappa_1^\dagger \star e^{2\rho} \star \kappa_2$$

It is not unique, we may take another one:

$$\widetilde{\langle \kappa_1, \kappa_2 \rangle} = \int d^2x \, e^{2\alpha\rho} \star \kappa_1^\dagger \star e^{2(1-\alpha)\rho} \star \kappa_2$$

which is, however, related to the original one by a change of the variables. The Dirac operator

$$\widehat{D} = i\gamma^\mu e^{-\rho} \star \left(\partial_\mu + \frac{1}{2}\omega_\mu \gamma_* \right)$$

contains an ambiguity related the ordering of $e^{-\rho}$ and ω. This ambiguity is fixed completely by the choice of the scalar product. ($\gamma_* = \gamma_1 \gamma_2$)

We know that the operator is essentially unique. Does it have any nice properties? The Lichnerowicz formula:

$$\widehat{D}^2 = \Delta_{\text{Spin}} + \frac{1}{8}\{e_a^\mu, e_b^\nu\} \star R_{\mu\nu}^{ab} + \frac{1}{8}\{e_a^\mu, e_b^\nu\} \star \epsilon^{ab}\eta_{cd} R_{\mu\nu}^{cd}\gamma_*$$

(stated in the conformal gauge only) Note: the terms on the r.h.s. are not necessarily real. This is because the left multiplication by a real function is not a hermitian operation!

Not so nice properties:

• The commutator $[\widehat{D}, f]$ is not a zero-order operator

• The integral $\int d^2x e^{2\rho} \star f$ is not a trace because $\int d^2x e^{2\rho} \star [f_1, f_2] \neq 0$.

This explains why this construction (dictated by physics!) was so far neglected by the mathematicians.

Scalar fields. We assume that in the conformal gauge massless minimally coupled scalar fields decouple from the geometry (as in the commutative case), so that the action reads

$$S_{\mathrm{m}} = \frac{1}{2}\int d^2x (\partial_\mu f^\dagger)(\partial_\mu f)$$

We take the same scalar product as before

$$\langle f_1, f_2 \rangle = \int d^2x \; f_1^\dagger \star e^{2\rho} \star f_2$$

By the change of the variables $\tilde{f} = e^\rho f$ we obtain a "trivial" scalar product

$$\langle \tilde{f}_1, \tilde{f}_2 \rangle_0 = \int d^2x \; \tilde{f}_1^\dagger \star \tilde{f}_2 = \int d^2x \; \tilde{f}_1^\dagger \tilde{f}_2$$

So that,

$$S_{\mathrm{m}} = \frac{1}{2}\int d^2x \tilde{f}^\dagger \star \Delta \star \tilde{f}, \quad \Delta = -e^{-\rho} \star \partial_\mu^2 e^{-\rho}$$

The measure for $\tilde{f}$ is the standard one, the path integral over $\tilde{f}$ is Gaussian, and

$$W_{\mathrm{m}} = \ln \det \Delta$$

Next:

- NC deformation of 2D dilaton gravities other than JT.
- Construction of a classical action for the matter fields. Study of classical effects near noncommutative black holes.
- More careful analysis of quantization, including the Hamiltonian constraint analysis, BRST, etc.
- More careful analysis of the spectral functions.
- Generalisations to the Kontsevich product (non-constant θ).

Thank You!