Supertwistorial N=2 String Field T

Olaf Lechtenfeld

Institut für Theoretische Physik, Universität Han

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Open $N{=}2$ strings on $\mathbb{R}^{2,2}$

Physical excitations: a single Lie-algebra valued massless sca

Non-zero tree-level amplitudes: $\langle \Phi^a(k_1)\, \Phi^b(k_2)\, \Phi^c(k_3) \rangle$

Dynamics from Leznov equation: $-\Box\Phi \;+\; \frac{1}{2}\,\partial_1{}^{\dot\alpha}\Phi\,\partial_{1\dot\alpha}\Phi$

This is the Yang-Mills self-duality condition in light-cone gauge

$A_{1\dot\alpha} = 0$ and $A_{2\dot\alpha} = \partial_{1\dot\alpha}\Phi \implies F_{\alpha\beta} \;\equiv\; \partial_{(\alpha}{}^{\dot\gamma} A_{\beta)}$

Chalmers-Siegel action with multiplier field $(G^{\alpha\beta}) = (0, 0, \widetilde{\Phi}$

$$S_{\mathrm{CS}} \;=\; \int \mathrm{tr}\; \widetilde{\Phi}\,\Big\{-\Box\Phi \;+\; \tfrac{1}{2}\,\partial_1{}^{\dot\alpha}\Phi\,\partial_{1\dot\alpha}\Phi\Big\} \;=\; \int \mathrm{tr}\; G$$

$\boxed{\text{Topological string on } \mathbb{R}^{2,2}}$

NSR formulation: left-moving $N{=}2$ superconformal algebra

$$T \;=\; \partial_z X^{\alpha\dot\beta}\partial_z X_{\alpha\dot\beta} + \psi^{\dot\alpha\ddot\beta}\partial_z\psi_{\dot\alpha\ddot\beta}$$

$$G^{1\ddot 2} \;=\; \psi^{\dot\gamma\ddot 2}\partial_z X^{1}{}_{\dot\gamma} \qquad\qquad G^{2\ddot 1} \;=\; \psi^{\dot\gamma\ddot 1}\partial_z X$$

$$J^{\ddot 1\ddot 2} \;=\; \psi^{\dot\gamma\ddot 1}\psi_{\dot\gamma}{}^{\ddot 2}$$

Berkovits-Vafa formulation: extension to small $N{=}4$ supercon

$$T \;=\; \partial_z X^{\alpha\dot\beta}\partial_z X_{\alpha\dot\beta} + \psi^{\dot\alpha\ddot\beta}\partial_z\psi_{\dot\alpha\ddot\beta}$$

$$G^{\alpha\ddot\beta} \;=\; \psi^{\dot\gamma\ddot\beta}\partial_z X^{\alpha}{}_{\dot\gamma}$$

$$J^{\ddot\alpha\ddot\beta} \;=\; \psi^{\dot\gamma\ddot\alpha}\psi_{\dot\gamma}{}^{\ddot\beta}$$

Twisting: $\quad G_\alpha{}^{\ddot 1} =: \bar D_\alpha \quad$ become two spin-one BRST-like

$$(\bar D_1)^2 \;=\; 0 \;=\; (\bar D_2)^2 \qquad \text{and} \qquad \{\bar D_1\,,\,\bar D_2\}$$

Covariant $N{=}2$ string field theory

We have switched: $\partial_{\alpha\dot\beta} \equiv \frac{\partial}{\partial x^{\alpha\dot\beta}} \;\hat{=}\; \partial_z X_{\alpha\dot\beta} \qquad \longrightarrow$

Lie-algebra valued string fields: 2 fermionic $A_\alpha[X,\psi]$ and 3 b

Berkovits-Siegel action: $S_{\mathsf{BS}} \;=\; \langle\, \mathsf{tr}\; G^{\alpha\beta}\, F_{\alpha\beta}(A)\,\rangle$

String field product = Witten's star product $\langle \ldots \rangle = \int$

$$F_{\alpha\beta}(A) \;:=\; \bar{D}_\alpha A_\beta \;+\; \bar{D}_\beta A_\alpha \;+\; \{A_\alpha\,,\, A_\beta$$

String field equations of motion:

$$F_{\alpha\beta} \;=\; 0 \qquad \text{and} \qquad \bar{D}_\alpha G^{\alpha\beta} \;+\; [A_\alpha\,,\, G^{\alpha\beta}]$$

Integrability of $N{=}2$ string field theor

Spectral parameter $\zeta \in \dot{\mathbb{C}} \;=\; \bar{H}_+ \cup \bar{H}_- \;=\; \{\mathrm{Im}\zeta \geq 0\} \cup$

Linear system for $\Psi_\pm[X,\psi,\zeta]$ holomorphic in $\bar{H}_\pm$:

$$(\bar{D}_2 - \zeta\bar{D}_1 + A_2 - \zeta A_1)\,\Psi_\pm \;=\; 0 \qquad \Longrightarrow \qquad (\bar{D}_2 - \zeta\bar{D}_1)$$

Conmpatibility condition:

$$(\bar{D}_2 - \zeta\bar{D}_1 + A_2 - \zeta A_1)^2 \;=\; 0 \qquad \Longleftrightarrow \qquad \bar{D}_\alpha A_\beta + \bar{D}_\beta A_\alpha$$

Chern-Simons form: $\bar{D} := \bar{D}_2 - \zeta\bar{D}_1$ and $A :=$

$$F_{\alpha\beta} \;=\; 0 \qquad \Longleftrightarrow \qquad (\bar{D} + A)^2 \;=\; 0 \qquad \Longleftrightarrow \qquad \bar{D}$$

Gauge fixing:

$$A_1 = 0 \qquad \text{and} \qquad A_2 = \bar{D}_1\Phi \qquad \Longrightarrow \qquad \bar{D}_2\bar{D}_1\Phi +$$

Asymptotics: $\Psi_\pm(\zeta{\to}\infty) \;=\; 1 \;+\; \zeta^{-1}\Phi \;+\; O(\zeta^{-2})$

Maximally supersymmetric extensio

Self-dual $N{=}4$ SYM multiplet: $(A_\alpha\,,\,\tilde\chi_i\,,\,\phi^{ij}\,,\,\chi^{\alpha i}\,,\,G^{\alpha\beta})$

Berkovits-Siegel action:

$$\widehat{S}_{\mathsf{BS}}\;=\;\langle\,\mathrm{tr}\Big(G^{\alpha\beta}\,F_{\alpha\beta}\;+\;\chi^{\alpha i}\,\nabla_\alpha\tilde\chi_i\;+\;\tfrac{1}{4}\phi^{ij}\,\nabla_\alpha\nabla^\alpha\phi_{ij}$$

with $\nabla_\alpha B\;:=\;\bar D_\alpha B+A_\alpha B-(-1)^{|B|}BA_\alpha$ and

BS equations of motion:

$$F_{\alpha\beta}\;\equiv\;\bar D_\alpha A_\beta\;+\;\bar D_\beta A_\alpha\;+\;\{A_\alpha\,,A_\beta\}\;=$$
$$\nabla_\alpha\tilde\chi_i\;=$$
$$\nabla_\alpha\chi^{i\alpha}\;+\;[\phi^{ij}\,,\tilde\chi_j]\;=$$
$$\nabla_\alpha\nabla^\alpha\phi_{ij}\;+\;[\tilde\chi_i\,,\tilde\chi_j]\;=$$
$$\nabla_\alpha G^{\alpha\beta}\;+\;[\tilde\chi_i\,,\chi^{i\beta}]\;+\;[\nabla^\beta\phi_{ij}\,,\phi^{ij}]\;=$$

BRST operator in twistor space

Notation: $(\zeta_\alpha) = \begin{pmatrix} 1 \\ \zeta \end{pmatrix} \qquad (\zeta^\alpha) = \begin{pmatrix} -\zeta \\ 1 \end{pmatrix} \qquad (\bar{\zeta}^\alpha) = \Bigg($

Twistor space of $\mathbb{R}^{2,2}$ is $\mathcal{P}^3 := \mathbb{R}^4 \times (H_+ \cup H_-) = \mathbb{R}^4 \times ($

Recall $\bar{D} = \zeta^\alpha \bar{D}_\alpha = \partial_z(\zeta^\alpha X_{\alpha\dot{\beta}})\, \psi^{\dot{\beta}\ddot{1}}$ is type (0,1

General type (0,1) vector field on $\mathcal{P}^3$: $Q := \bar{D} + \bar{\partial} =$

$\{\mathrm{d}\bar{\zeta}, \psi^{\dot{\alpha}\ddot{\beta}}\} = 0 \quad \Longrightarrow \quad Q^2 = 0\,.$ We take Q as our BRST

Supertwistor space $\mathcal{P}^{3|4}$: add four Grassmann variables $\theta^i \in$

Field theory for $N{=}2$ strings on supertwisto

Consider fermionic Lie-algebra valued string fields $\quad \mathcal{A}\Big[X(\sigma)$

Note: The ζ, θ^i coordinates of $\mathcal{P}^{3|4}$ are world-sheet *cons*

Assume splitting as for Q: $\quad \mathcal{A} \;=\; A[X,\psi;\zeta,\theta^i] \;+\; a[X,$

with $\quad A \in \mathcal{O}(1) \quad$ and $\quad a = a_{\bar\zeta}\,\mathrm{d}\bar\zeta \in \mathcal{O}(0) \quad$ gauging $\bar{D}$ a

Cubic Chern-Simons type action:

$$S_3 \;=\; \int \mathrm{d}\zeta \int \mathrm{d}^4\theta \;\langle\, \mathrm{tr}\Big(\mathcal{A}\, Q\mathcal{A} \;+\; \tfrac{2}{3}\mathcal{A}^3\Big)\,\rangle$$

$$=\; \int \mathrm{d}\zeta \int \mathrm{d}^4\theta \;\langle\, \mathrm{tr}\Big(A\,\bar\partial A \;+\; 2\,A\,\bar{D}a \;+\; 2\,A^2 a\Big)$$

$$=\; \int \mathrm{d}\zeta\wedge\mathrm{d}\bar\zeta \int \mathrm{d}^4\theta \;\langle\, \mathrm{tr}\Big(-A\,\partial_{\bar\zeta}A \;+\; 2\,A\,\bar{D}a_{\bar\zeta} \;+$$

because the measure $\quad \mathrm{d}\zeta\,\mathrm{d}^4\theta \;=\; \mathrm{d}\zeta\,\mathrm{d}\theta^1\,\mathrm{d}\theta^2\,\mathrm{d}\theta^3\,\mathrm{d}\theta^4 \;\in \mathcal{O}$

Equations of motion: $\quad \bar{D}a + \bar\partial A + \{A\,,a\} \;=\; 0 \quad$ and

Component analysis

Recall that $A \in \mathcal{O}(1)$ and $a_{\bar\zeta} \in \bar{\mathcal{O}}(-2)$.

Since $\zeta^\alpha \in \mathcal{O}(1)$, $\bar\zeta^\alpha \in \bar{\mathcal{O}}(1)$, $\theta^i \in \mathcal{O}(1)$ and nilpoter

the dependence of A and $a_{\bar\zeta}$ on θ^i, ζ and $\bar\zeta$ is determined (up

$$A \;=\; \zeta^\alpha A_\alpha + \theta^i \tilde\chi_i \;+\; \tfrac{\nu}{2}\theta^i\theta^j\bar\zeta^\alpha\phi_{\alpha ij} \;+\; \nu^2\theta_i\bar\zeta^\alpha\bar\zeta^\beta\chi^i_{\alpha\beta} \;+$$

$$a_{\bar\zeta} \;=\; \tfrac{\nu^2}{2}\theta^i\theta^j\phi_{ij} \;+\; \nu^3\theta_i\bar\zeta^\alpha\chi^i_\alpha \;+\; \nu^4\theta\,\bar\zeta^\alpha\bar\zeta^\beta G_{\alpha\beta}$$

with $\theta^i\theta^j\theta^k =: \epsilon^{ijk\ell}\,\theta_\ell$ and $\theta^i\theta^j\theta^k\theta^\ell =: \epsilon^{ijk\ell}\,\theta$

Substituting this expansion into the equations of motion yields

$\phi_{\alpha ij} = -\nabla_\alpha\phi_{ij}$ and $\chi^i_{\alpha\beta} = \tfrac{1}{2}\nabla_{(\alpha}\chi^i_{\beta)}$ and G

as well as the BS eq's of motion for $A_\alpha\,,\ \tilde\chi_i\,,\ \phi_{ij}\,,\ \chi^i_\alpha\,,\ G_{\alpha\beta}$ e

Also directly: $S_3[A,a] \;\propto\; \hat S_{\mathrm{BS}}[A_\alpha, \tilde\chi_i, \phi_{ij}, \chi^i_\alpha, G_{\alpha\beta}]$ —

Bosonic truncation

Also *non*-supersymmetric self-dual YM via $N{=}2$ SFT in supe

Yes indeed! Truncate $\int \mathrm{d}^4\theta \begin{Bmatrix} \theta^i \\ \theta^i\theta^j \\ \theta^i\theta^j\theta^k \end{Bmatrix} \mathcal{A} \;=\; 0$

Then string fields depend on $\{\theta^i\}$ only in the combination $\theta =$

This is equivalent to putting $\tilde{\chi}_i \;=\; \phi_{ij} \;=\; \chi^i_\alpha \;=\; 0$

and keeping only A_α and $G^{\alpha\beta}$ in the action, hence: $\widehat{S}_{\mathrm{BS}}$

The expansion

$$A \;=\; \zeta^\alpha\, A_\alpha[X,\psi] \;+\; \theta\,\nu^3\bar{\zeta}^\alpha\bar{\zeta}^\beta\bar{\zeta}^\gamma\, G_{\alpha\beta\gamma}[$$

$$a \;=\; \theta\,\nu^4\bar{\zeta}^\alpha\bar{\zeta}^\beta\, G_{\alpha\beta}[X,\psi]\,\mathrm{d}\bar{\zeta}$$

yields indeed

$$S_3 \;=\; \int \mathrm{d}\zeta \int \mathrm{d}^4\theta\, \langle\, \mathrm{tr}\left(\mathcal{A}\,Q\mathcal{A} \;+\; \tfrac{2}{3}\mathcal{A}^3\right)\rangle \;\propto\; \langle\, \mathrm{tr}\, G^{\alpha\beta}\, F_{\alpha}$$

String zero modes reproduce twistor description of ($N{=}4$ or I

Beyond tree level

All considered actions are *one-loop exact*!   For example, i

$$S_{\text{CS}} \;=\; \int \text{tr}\, \widetilde{\Phi} \left\{ -\Box\Phi + \tfrac{1}{2}\,\partial_1{}^{\dot\alpha}\Phi\,\partial_{1\dot\alpha}\Phi \right\} \;=\; \int \text{tr}\; G$$

the multiplier field $\widetilde{\Phi}$ or $G^{\alpha\beta}$ acts as a loop counting parame

Any amplitude at L loops must carry precisely $1{-}L$ external n

$\longrightarrow$ all amplitudes vanish beyond one loop

Self-dual fields carry a single helicity $(+)$, multiplier fields the

Capture gluonic MHV amplitudes $\langle - + \cdots + \rangle|_{\text{tree}}$ and $\langle$

Coupling to (self-dual) gravity

Open plus closed $N{=}2$ strings have two physical excitations:

a massless Lie-algebra valued scalar Φ and a massless sca

The tree-level amplitudes yield the following dynamics:

$$-\Box\Phi \;+\; \tfrac{1}{2}\,\partial_1{}^{\dot\alpha}\Phi\,\partial_{1\dot\alpha}\Phi \;+\; \kappa\,\partial_1{}^{\dot\alpha}\partial_1{}^{\dot\beta}\Omega\,\partial_{1\dot\alpha}\partial_{1\dot\beta}\Phi$$

$$-\Box\Omega \;+\; \tfrac{\kappa}{2}\,\partial_1{}^{\dot\alpha}\partial_1{}^{\dot\beta}\Omega\,\partial_{1\dot\alpha}\partial_{1\dot\beta}\Omega \;+\; O(\alpha')$$

This is the Einstein-Yang-Mills self-duality condition in light-co

$$A_{2\dot\alpha} \;=\; \partial_{1\dot\alpha}\Phi \qquad \text{and} \qquad \omega_{2\dot\alpha\dot\beta\dot\gamma} \;=\; \partial_{1\dot\alpha}\partial_{1\dot\beta}\partial_{1\dot\gamma}\Omega$$

$$F_{\dot\alpha\dot\beta} \;=\; \partial_{1\dot\alpha}\partial_{1\dot\beta}\Phi \qquad \text{and} \qquad C_{\dot\alpha\dot\beta\dot\gamma\dot\delta} \;=\; \partial_{1\dot\alpha}\partial_{1\dot\beta}\partial_{1\dot\gamma}\partial_{1\dot\delta}\Omega$$

$$F_{\alpha\beta} \;=\; 0 \qquad \text{and} \qquad R_{\alpha\beta} \;\equiv\; \epsilon_{\alpha\gamma}\epsilon_{\beta\delta}\,R\,\Gamma^{\gamma\delta} + C_{\alpha\beta\gamma\delta}\,\Gamma^{\gamma\delta} +$$

Can we repeat the same game for self-dual gravity?