DARK ENERGY

Dark matter and dark energy – two entities in the Universe seen by their gravitational interaction only

DM – non-relativistic
gravitationally clustered

DE – relativistic
unclustered

Definition through equations

I. DM – through the (generalized) Poisson equation

$$\frac{\Delta \Phi}{a^2} = 4\pi G(\rho - \rho_0(t))$$

$\Phi(\vec{r})$ is measured using the motion of 'test particles' in it

a) Stars in galaxies → rotation curves

b) Galaxies → peculiar velocities

c) Hot gas in clusters → X-ray profiles

d) Photons → gravitational lensing (strong and weak)

Result: DM is non-relativistic ($p \ll E$), collisionless, and has the same spatial distribution as visible matter for $L \gtrsim 1$ Mpc

$$b \simeq 1, \quad \Omega_{m,tot} = 0.28 \pm 0.03 \qquad (26)$$
$$\Omega_{bar} = 0.046 \pm 0.003$$

II. DE – through relativistic gravitational field equations in the Einsteinian form

$$\frac{1}{8\pi G}\left(R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R\right) = \underbrace{T_{\mu(vis)}^\nu + T_{\mu(DM)}^\nu}_{\text{approximately dust-like}} + T_{\mu(DE)}^\nu$$

$G = G_0 = const$ – the Newton gravitational constant measured in laboratory

$T^\nu_{\mu(DE);\nu} = 0$ (in the absence of decay of DM particles into DE)

Applications of the definition of $T^{\nu}_{\mu(DE)}$

1. To the FRW background

$$\varepsilon_{DE} \equiv T^{0}_{0(DE)} = \varepsilon_{DE}(z) \qquad z = z(t) = \frac{a_0}{a(t)} - 1$$

$$p_{DE} = -T^{\alpha}_{\alpha(DE)} = p_{DE}(z) \qquad \text{(no summation on } \alpha\text{)}$$

$$\dot{\varepsilon}_{DE} + 3H(\varepsilon_{DE} + p_{DE}) = 0 \qquad H \equiv \frac{\dot{a}}{a}$$

1 function

2. To the evolution of density perturbations in the matter component (baryons + DM) at small scales

$\left(\frac{\delta\rho}{\rho}\right)_m (z)$ – second observational function

Main up-to-date result

In the zero approximation (~10% accuracy)

$$\varepsilon_{DE}(z) = \varepsilon_0 = \text{const} \qquad \Omega_{DE} = 1 - \Omega_m$$

$$p_{DE} = -\varepsilon_0, \quad T^{\nu}_{\mu(DE)} = \varepsilon_0 \delta^{\nu}_{\mu}$$

$$\rho_0 = \frac{\varepsilon_0}{c^2} = 6.44 \cdot 10^{-30} \ \frac{\Omega_{DE}}{0.7} \cdot \left(\frac{H_0}{70}\right)^2 \ \text{g cm}^{-3}$$

$$\frac{G^2 \hbar \varepsilon_0}{c^7} = 1.25 \cdot 10^{-123} \cdot \overbrace{(\ldots\downarrow\ldots)}$$

Present matter content of the Universe

(in terms of the critical density

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} \simeq 10^{-29}\ g/cm^3 , \quad \Omega_i = \frac{\rho_i}{\rho_{crit}})$$

1. Baryons (p, n) and leptons (e) — $(4.5 \pm 0.5)\,\%$ — No antimatter
2. Photons (γ) — $5 \cdot 10^{-5}$
3. Neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ — $\leq 1\%$ — $(\sum_i m_{\nu i} < 0.6\,eV)$
   $\sum_i m_{\nu i}(eV) = 94\,\Omega_\nu h^2$
4. Non-relativistic non-baryonic dark matter — $\simeq 23\%$
5. Dark energy — $\simeq 72\%$

$\left.\begin{array}{l} 4. \\ 5. \end{array}\right\}$ Not known from laboratory experiments

SCM: $\Lambda CDM + (\mathcal{K}=0) + \left(\begin{smallmatrix} n_S = 1 \\ \text{adiabatic} \end{smallmatrix}\right)$

$\sim 10\%$ accuracy

4 main fundamental constants and related theories:

1. $$\frac{\hbar^3 \varepsilon_\Lambda}{c^5 M_{Pl}^4} = \frac{G^2 \hbar \varepsilon_\Lambda}{c^7} = 1.25\cdot 10^{-123}\cdot\frac{\Omega_\Lambda}{0.7}\cdot\left(\frac{H_0}{70}\right)^2$$

   Theory of a cosmological constant (dark energy)

2. $$\frac{\varepsilon_b}{\varepsilon_m} = 0.156\cdot\frac{\Omega_b h^2}{0.023}\cdot\left(\frac{70}{H_0}\right)^2\cdot\frac{0.3}{\Omega_m} \qquad h = \frac{H_0}{100}$$

   $$\varepsilon_m = \varepsilon_b + \varepsilon_{CDM}$$

   Theory of dark non-baryonic matter

3. $$\frac{n_b}{n_\gamma} = 6.25\cdot 10^{-10}\cdot\frac{\Omega_b h^2}{0.023}\cdot\left(\frac{2.73}{T_\gamma}\right)^3$$

   Theory of baryogenesis

4. $$\Phi_0 = 2.94\cdot 10^{-5}\cdot\left(\frac{\Delta_R^2}{2.4\cdot 10^{-9}}\right)^{1/2}$$

   $$\langle \Phi_{in}^2 \rangle = \Phi_0^2 \int \frac{dk}{k}$$ at the MD stage

   Theory of initial conditions – inflation

Table 6. Cosmological Parameter Summary

Description	Symbol	*WMAP*-only	*WMAP*+BAO+SN
		Parameters for Standard ΛCDM Model [a]	
Age of universe	t_0	13.69 ± 0.13 Gyr	13.73 ± 0.12 Gyr
Hubble constant	H_0	$71.9^{+2.6}_{-2.7}$ km/s/Mpc	70.1 ± 1.3 km/s/Mpc
Baryon density	Ω_b	0.0441 ± 0.0030	0.0462 ± 0.0015
Physical baryon density	$\Omega_b h^2$	0.02273 ± 0.00062	0.02265 ± 0.00059
Dark matter density	Ω_c	0.214 ± 0.027	0.233 ± 0.013
Physical dark matter density	$\Omega_c h^2$	0.1099 ± 0.0062	0.1143 ± 0.0034
Dark energy density	Ω_Λ	0.742 ± 0.030	0.721 ± 0.015
Curvature fluctuation amplitude, $k_0 = 0.002$ Mpc^{-1} [b]	$\Delta^2_{\mathcal{R}}$	$(2.41 \pm 0.11) \times 10^{-9}$	$(2.457^{+0.092}_{-0.093}) \times 10^{-9}$
Fluctuation amplitude at $8h^{-1}$ Mpc	σ_8	0.796 ± 0.036	0.817 ± 0.026
$l(l+1)C^{TT}_{220}/2\pi$	C_{220}	5756 ± 42 μK^2	5748 ± 41 μK^2
Scalar spectral index	n_s	$0.963^{+0.014}_{-0.015}$	$0.960^{+0.014}_{-0.013}$
Redshift of matter-radiation equality	$z_{\rm eq}$	3176^{+151}_{-150}	3280^{+88}_{-89}
Angular diameter distance to matter-radiation eq. [c]	$d_A(z_{\rm eq})$	14279^{+186}_{-189} Mpc	14172^{+141}_{-139} Mpc
Redshift of decoupling	z_*	1090.51 ± 0.95	$1091.00^{+0.72}_{-0.73}$
Age at decoupling	t_*	380081^{+5843}_{-5841} yr	375938^{+3148}_{-3115} yr
Angular diameter distance to decoupling [c,d]	$d_A(z_*)$	14115^{+188}_{-191} Mpc	14006^{+142}_{-141} Mpc
Sound horizon at decoupling [d]	$r_s(z_*)$	146.8 ± 1.8 Mpc	145.6 ± 1.2 Mpc
Acoustic scale at decoupling [d]	$l_A(z_*)$	$302.08^{+0.83}_{-0.84}$	$302.11^{+0.84}_{-0.82}$
Reionization optical depth	τ	0.087 ± 0.017	0.084 ± 0.016
Redshift of reionization	$z_{\rm reion}$	11.0 ± 1.4	10.8 ± 1.4
Age at reionization	$t_{\rm reion}$	427^{+88}_{-65} Myr	432^{+90}_{-67} Myr
		Parameters for Extended Models [e]	
Total density [f]	$\Omega_{\rm tot}$	$1.099^{+0.100}_{-0.085}$	1.0052 ± 0.0064
Equation of state [g]	w	$-1.06^{+0.41}_{-0.42}$	$-0.972^{+0.061}_{-0.060}$
Tensor to scalar ratio, $k_0 = 0.002$ Mpc^{-1} [b,h]	r	< 0.43 (95% CL)	< 0.20 (95% CL)
Running of spectral index, $k_0 = 0.002$ Mpc^{-1} [b,i]	$dn_s/d\ln k$	-0.037 ± 0.028	$-0.032^{+0.021}_{-0.020}$
Neutrino density [j]	$\Omega_\nu h^2$	< 0.014 (95% CL)	< 0.0065 (95% CL)
Neutrino mass [j]	$\sum m_\nu$	< 1.3 eV (95% CL)	< 0.61 eV (95% CL)
Number of light neutrino families [k]	$N_{\rm eff}$	> 2.3 (95% CL)	4.4 ± 1.5

[a]The parameters reported in the first section assume the 6 parameter ΛCDM model, first using *WMAP* data only (Dunkley et al. 2008), then using *WMAP*+BAO+SN data (Komatsu et al. 2008).

[b]$k = 0.002$ Mpc^{-1} $\longleftrightarrow$ $l_{\rm eff} \approx 30$.

[c]Comoving angular diameter distance.

Two possible forms and interpretations of DE

1. Physical

New non-gravitational field of matter

Its proper place – in the rhs of eqn.

2. Geometrical

Depends on the Riemann tensor of our 4D or additional dimensions

Its proper place – in the lhs of eqn.
Gravity is modified

No absolute border between these 2 cases

Λ – intermediate case

Another intermediate case (but closer to geometrical DE)

non-minimally coupled scalar field

$G_{eff} \neq const$

Comments about "cosmic coincidences" for Λ

1. Why small? Not known why so small, but all known dimensionless densities are very small

$$\rho_{DE} \sim \rho_{water}^{4/3} \sim m_\nu^4$$

2. Why now? Not an independent problem. Reduces to the first problem (plus relations between other fundamental constants) once "now" is defined in an objective way.

It is natural to use (weak) anthropic principle to define "now". However, in practice, very remote arguments are used.

Example. Let us, following Dicke, define $t_0 \sim t_{\substack{\text{active life} \\ \text{main sequence} \\ \text{star}}} \sim t_{\mathcal{G}\ell} \cdot \left(\frac{M_{\mathcal{G}\ell}}{m_p}\right)^3$

Then the "second coincidence problem" is reduced to the first one because of the empirical relation

$$\rho_{DE} \sim \left(\frac{m_p}{M_{\mathcal{G}\ell}}\right)^6$$

One constant — one problem ("coincidence")

Investigation of dark energy

I. From observations to theory

Reconstruction program (1998)

1) $H(z)$, $\varepsilon_{DE}(z)$
2) $q(z)$, $p_{DE}(z)$, $w_{DE}(z)$
3) $r(z)$, $\frac{dw_{DE}(z)}{dz}$

1. Inversion of classical cosmological tests $D_L(z) \to H(z)$
2. CMB (acoustic peaks spacing, ISW), BAO
3. $\left(\frac{\delta\rho}{\rho}\right)_m(z)$, $\Phi(z)$ from gravitational lensing, correlation of $\frac{\delta\rho}{\rho}$ with CMB

II. From theory to observations

Models (many of them!)
(qualitatively – the same as for inflation)

1. Fundamental constant Λ
2. Scalar field (with $m \sim 10^{-33}$ eV, $w_{DE} \geq -1$)
3. Geometrical DE = modified gravity (e.g., scalar-tensor and f(R) DE models)

Basic quantities in the reconstruction approach

Order	Geometrical	Physical
1	$H(z) \equiv \frac{\dot a}{a}$ $H(0) = H_0$	$\varepsilon_m = \frac{3H_0^2}{8\pi G} \cdot \Omega_{m0}(1+z)^3$ $\varepsilon_{DE} = \frac{3H^2}{8\pi G} - \varepsilon_m$
2	$q(z) \equiv -\frac{\ddot a a}{\dot a^2}$ $= -1 + \frac{d \ln H}{d \ln(1+z)}$ $q(0) = q_0$ For $\varepsilon_\Lambda = const$: $q(z) = -1 + \frac{3}{2}\Omega_m(z)$	$V(z);\ T(z) \equiv \frac{\dot\varphi^2}{2}$ $\Omega_V = \frac{8\pi G V}{3H^2};\ \Omega_T = \frac{8\pi G T}{3H^2}$ $\Omega_V = \frac{2-q}{3} - \frac{H_0^2}{2H^2}\Omega_{m0}(1+z)^3$ $\Omega_T = \frac{1+q}{3} - \frac{H_0^2}{2H^2}\Omega_{m0}(1+z)^3$
3	$r(z) \equiv \frac{\dddot a a^2}{\dot a^3}$ $r(0) = r_0$ For $\varepsilon_\Lambda \equiv const$: $r \equiv 1$	$\Pi(z) \equiv \dot\varphi V'$ $\Omega_\Pi = \frac{8\pi G \dot\varphi V'}{3H^3}$ $\Omega_\Pi = \frac{1}{3}\Big(r - 3q - 4 + \frac{9H_0^2}{2H^2}\Omega_{m0}(1+z)^3\Big)$

Derivative quantity :

$$w = \frac{T-V}{T+V} = \frac{2q-1}{3\left(1 - \frac{H_0^2}{H^2}\Omega_{m0}(1+z)^3\right)}$$

$w \geq -1$ - "normal" DE

$w < -1$ - "phantom" DE

Λ : $w \equiv -1$

CLASSICAL COSMOLOGICAL TESTS AND THEIR INVERSION (RECONSTRUCTION OF H(Z))

1. High-z supernovae test

$$D_L(z) = a_0(\eta_0 - \eta)(1+z), \qquad \eta = \int_0^t \frac{dt}{a(t)}$$

$$H(z) = \frac{da}{a^2 d\eta} = -(a_0\eta')^{-1} = \left[\left(\frac{D_L(z)}{1+z}\right)'\right]^{-1}$$

2. Angular size test

$$\theta(z) = \frac{d}{a(\eta)(\eta_0 - \eta)} = \frac{d(1+z)}{a_0(\eta_0 - \eta)}$$

$$H(z) = -(a_0\eta')^{-1} = \left[d\left(\frac{1+z}{\theta(z)}\right)'\right]^{-1}$$

3. Volume element test

$$\frac{dN}{dz\,d\Omega} \propto \frac{dV}{dz\,d\Omega} = a^3 r^2 \left|\frac{dr}{dz}\right| =$$

$$= a^3 (r_0 - r)^2 \left|\frac{dr}{dz}\right| = f_V(z)$$

$$f_V(z) = \frac{1}{(1+z)^3 H(z)} \left(\int_0^z \frac{dz}{H(z)}\right)^2$$

$$H^{-1}(z) = \frac{d}{dz}\left\{\left(3\int_0^z f_V(z')(1+z')^3 dz'\right)^{1/3}\right\}$$

4. Ages of old objects at high z

$$T(z) > t_i(z)$$

$$T(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')}$$

look-back time

$$H(z) = -\left((1+z)\frac{dT(z)}{dz}\right)^{-1}$$

5. High-z clustering tests

For $\lambda \ll \lambda_{\gamma,\psi} \sim R_h$:

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}\frac{C}{a^3}\delta = 0$$

May be more complicated for geometric dark energy

$$\frac{d}{dt} = aH\frac{d}{da} \qquad C = \Omega_m H_0^2 a_0^3$$

$$H^2(a) = \frac{3C}{\delta'^2 a^6}\int_0^a \delta\delta' a\,da$$

$$a = \frac{a_0}{1+z}$$

$$\frac{H^2(z)}{H_0^2} = 3\Omega_m \frac{(1+z)^2}{\left(\frac{d\delta}{dz}\right)^2}\int_z^\infty \delta\left|\frac{d\delta}{dz}\right|\frac{dz}{1+z} =$$

$$= \frac{(1+z)^2\delta'^2(0)}{\delta'^2(z)} - \frac{3\Omega_m(1+z)^2}{\delta'^2(z)}\int_0^z \delta\delta'\frac{dz}{1+z}$$

(A.S., 1998)

Determination of Ω_m and q_0 from $\delta(z)$:

$$\Omega_m = \frac{\delta'^2(0)}{3\left|\int_0^\infty \delta\delta'\frac{dz}{1+z}\right|}$$

The textbook expression

(Weinberg, Peebles, etc.)

$$\delta(z) \propto H(z) \int_z^\infty \frac{(1+z')\,dz'}{H^3(z')}$$

Theorem

It is valid if and only if

$$H^2(z) = C_1 + C_2(1+z)^2 + C_3(1+z)^3$$

$$\longleftrightarrow$$

The super-Hubble solution ($k \ll aH$)

$$\delta(z) \propto a\left(1 - \frac{H}{a}\int a\,dt\right)$$

Valid for subhorizon scales if

$$H^2(z) = C_1 + C_2(1+z)^3$$

$$\longleftrightarrow$$

From 2dF survey: $\left.\dfrac{d\ln\delta}{d\ln(1+z)}\right|_{z=0.15} = -0.51 \pm 0.11$

How to determine $\delta(z)$

a) Evolution of clustering with z

$$r_0(z)$$

b) Evolution of rich cluster abundance with z

$$n(\geqslant M)(z)$$

c) Weak gravitational lensing of galaxies and CMB

$$\Phi(z)$$

6. CMB tests

a) Spacing between acoustic peaks

$$R \equiv \sqrt{\Omega_{m0}}\, H_0 \int_0^{z_{rec}} \frac{dz}{H(z)} = 1.715 \pm 0.021$$

Precise but degenerate test (WMAP5)

b) Correlation between $\frac{\Delta T}{T}$ and LSS (due to the ISW effect)

7. Sakharov oscillations in $P_0(k)$ (BAO)

$$\sqrt{\Omega_{m0}} \left(\frac{H_0}{H(z_1)}\right)^{1/3} \left(\frac{1}{z_1} \int_0^{z_1} dz \cdot \frac{H_0}{H(z)}\right)^{2/3} = 0.469 \left(\frac{n_s}{0.98}\right)^{-0.35} \pm 0.017$$

$z_1 = 0.35,\ 0.20$

(Eisenstein et al., 2005)

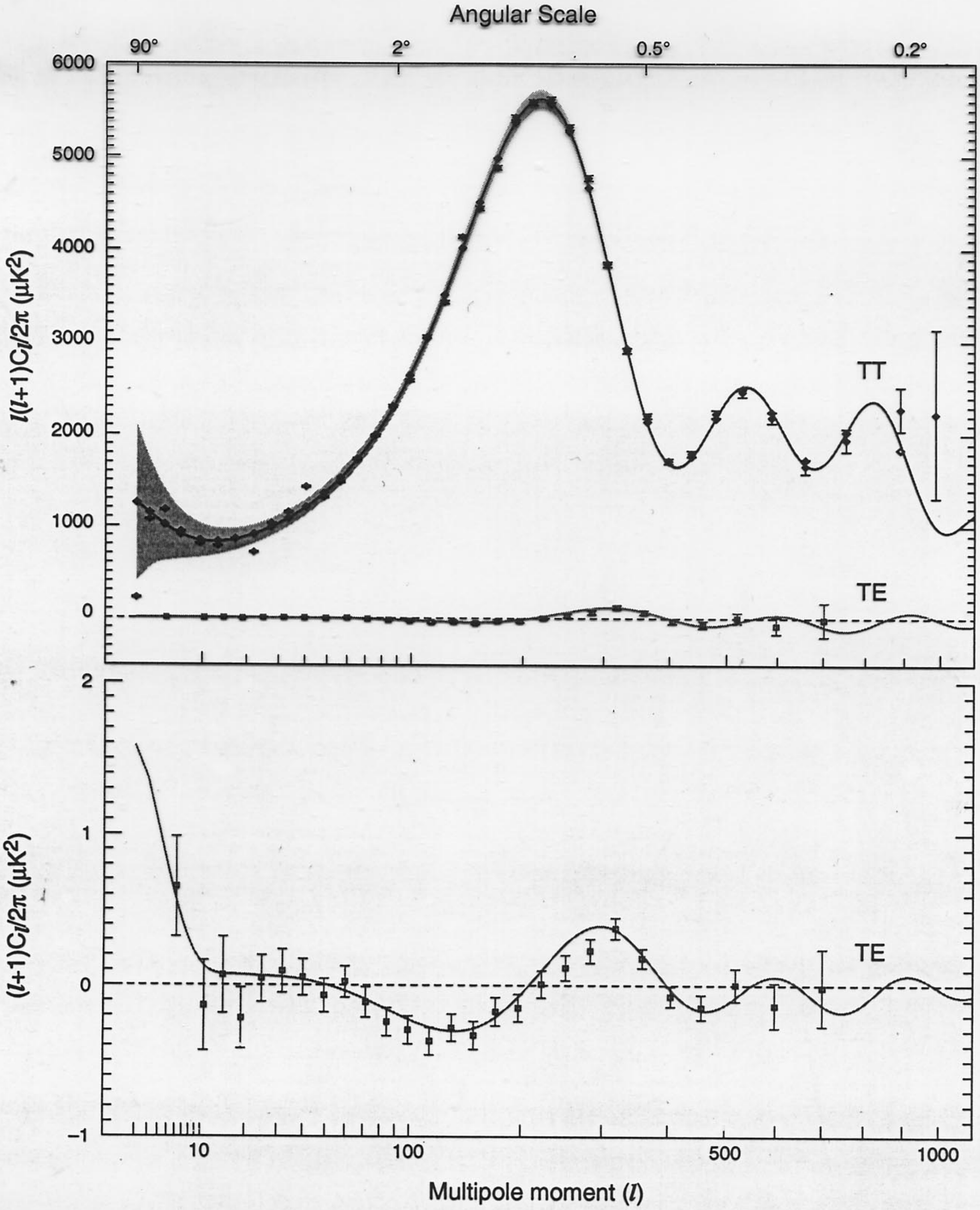


Fig. 22.— Angular power spectra C_l^{TT} & C_l^{TE} from the three-year *WMAP* data. *top*: The TT data are as shown in Figure 16. The TE data are shown in units of $l(l+1)C_l/2\pi$, on the same scale as the TT signal for comparison. *bottom*: The TE data, in units of $(l+1)C_l/2\pi$. This updates Figure 12 of Bennett et al. (2003b).

M. Tegmark et al., ApJ 606, 702 (2004)

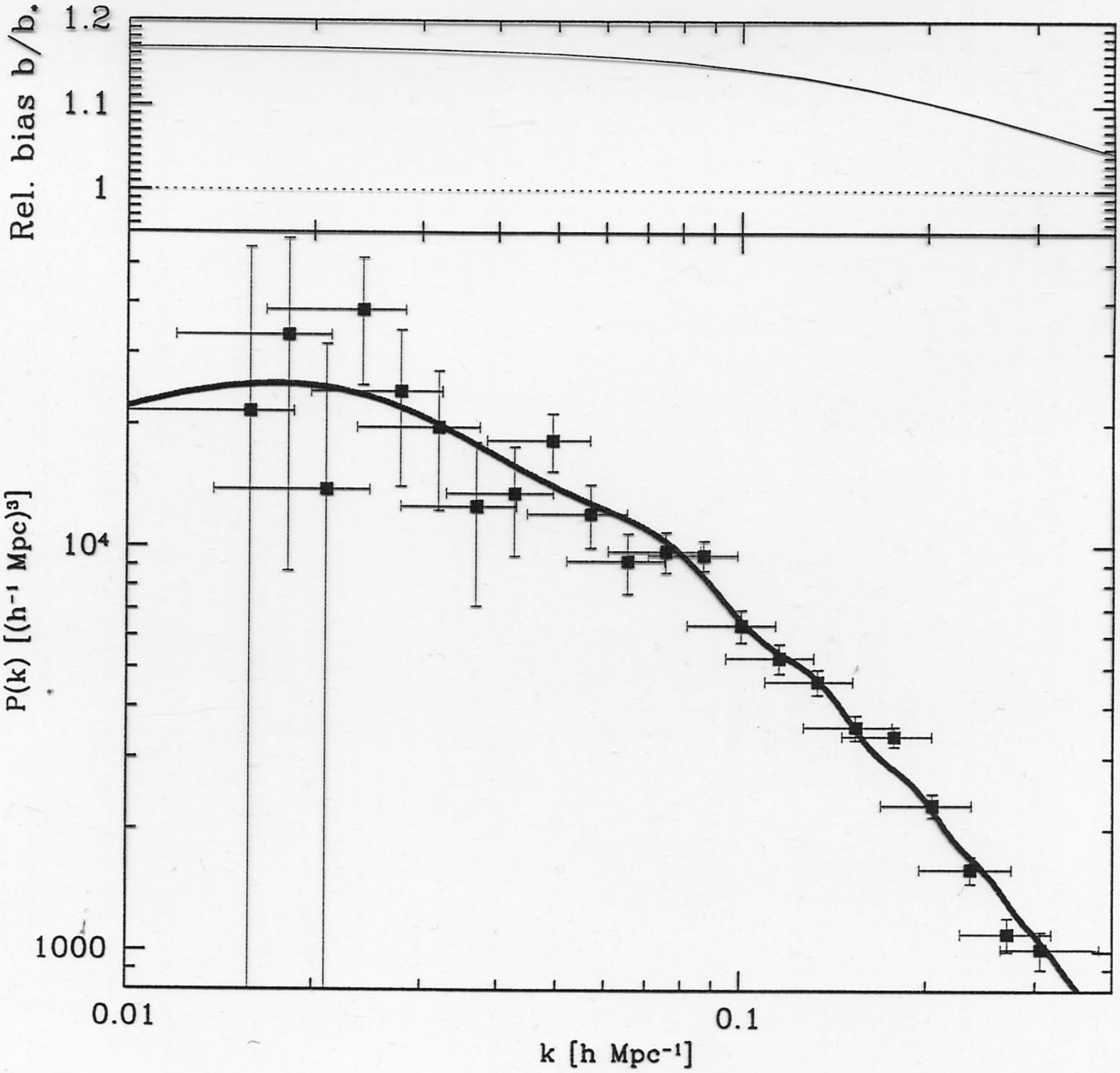


FIG. 22.— The decorrelated real-space galaxy-galaxy power spectrum using the modeling method is shown (bottom panel) for the baseline galaxy sample assuming $\beta = 0.5$ and $r = 1$. As discussed in the text, uncertainty in β and r contribute to an overall calibration uncertainty of order 4% which is not included in these error bars. To remove scale-dependent bias caused by luminosity-dependent clustering, the measurements have been divided by the square of the curve in the top panel, which shows the bias relative to L_* galaxies. This means that the points in the lower panel can be interpreted as the power spectrum of L_* galaxies. The solid curve (bottom) is the best fit linear ΛCDM model of Section 5.

D. E. Eisenstein et al., astro-ph/0501171

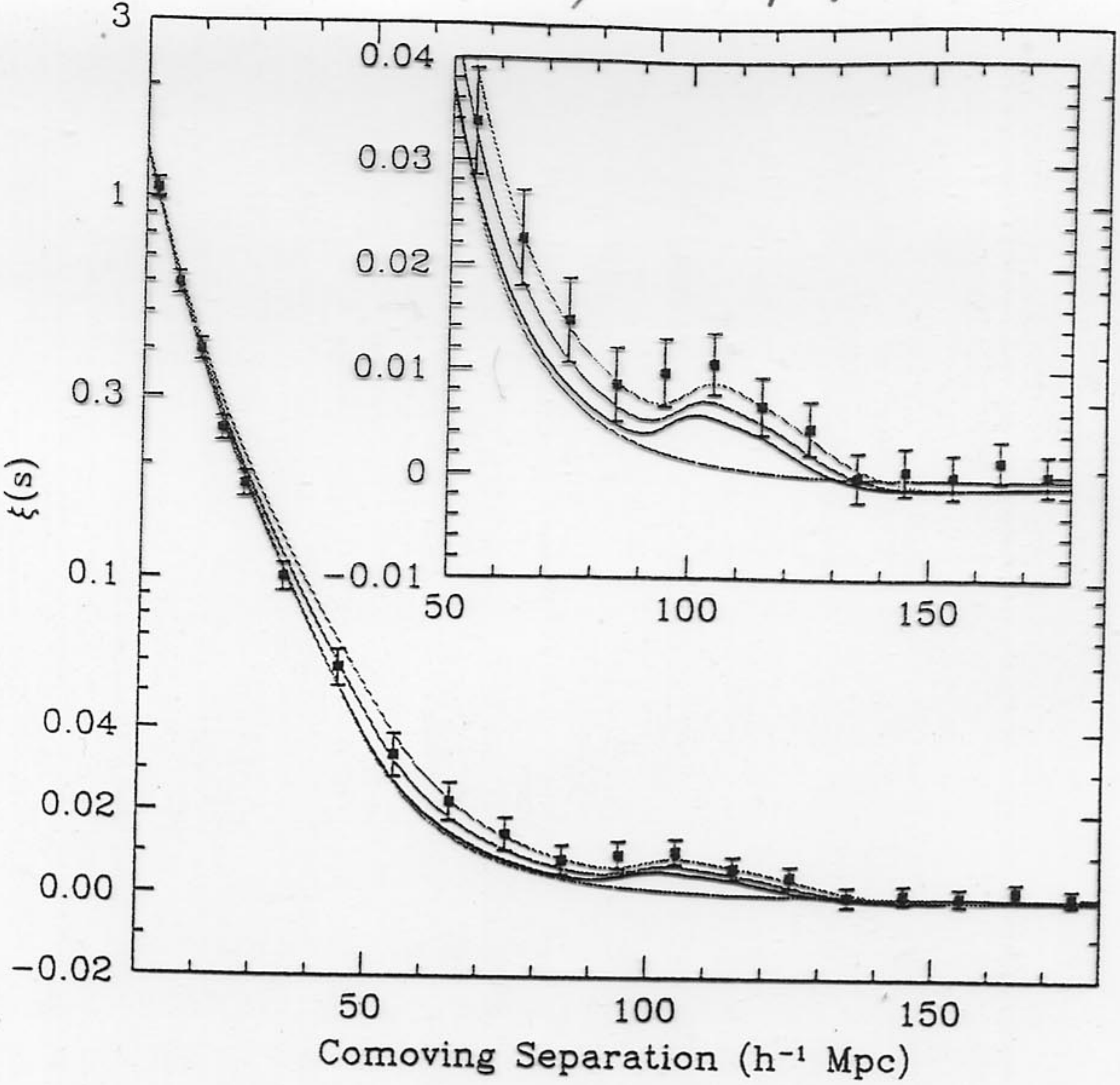


FIG. 2.— The large-scale redshift-space correlation function of the SDSS LRG sample. The error bars are from the diagonal elements of the mock-catalog covariance matrix; however, the points are correlated. Note that the vertical axis mixes logarithmic and linear scalings. The inset shows an expanded view with a linear vertical axis. The models are $\Omega_m h^2 = 0.12$ (top, green), 0.13 (red), and 0.14 (bottom with peak, blue), all with $\Omega_b h^2 = 0.024$ and $n = 0.98$ and with a mild non-linear prescription folded in. The magenta line shows a pure CDM model ($\Omega_m h^2 = 0.105$), which lacks the acoustic peak. It is interesting to note that although the data appears higher than the models, the covariance between the points is soft as regards overall shifts in $\xi(s)$. Subtracting 0.002 from $\xi(s)$ at all scales makes the plot look cosmetically perfect, but changes the best-fit χ^2 by only 1.3. The bump at $100h^{-1}$ Mpc scale, on the other hand, is statistically significant.

$$q_0 = -1 + \left.\frac{d\ln H}{d\ln(1+z)}\right|_{z=0}$$

$$\Lambda = \mathrm{const} \longrightarrow H^2(z) = H_0^2\left(1 - \Omega_m + \Omega_m (1+z)^3\right)$$

$$\longrightarrow \quad q_0 = \frac{3}{2}\Omega_m - 1$$

II. Reconstruction of $V(\varphi)$ from $H(a)$

$$\begin{cases} 8\pi G V = aH\dfrac{dH}{da} + 3H^2 - \dfrac{3}{2}\Omega_m H_0^2 \cdot \left(\dfrac{a_0}{a}\right)^3 \\ 4\pi G\, a^2 H^2 \left(\dfrac{d\varphi}{da}\right)^2 = -aH\dfrac{dH}{da} - \dfrac{3}{2}\Omega_m H_0^2 \left(\dfrac{a_0}{a}\right)^3 \end{cases}$$

Necessary condition $(\varepsilon_{DE} + p_{DE} \geq 0)$

$w_{DE} \equiv \dfrac{p_{DE}}{\varepsilon_{DE}} \geq -1$

$$\frac{dH^2}{dz} \geq 3\Omega_m H_0^2 (1+z)^2$$

$$H^2 \geq H_0^2\left(1 + \Omega_m (1+z)^3 - \Omega_m\right)$$

In particular: $\quad q_0 \geq \frac{3}{2}\Omega_m - 1$

No such a condition in case of scalar-tensor gravity

Example of the reconstruction

Let $\Lambda \propto a^{-q}$ or $p_\Lambda = \left(-1 + \frac{q}{3}\right)\varepsilon_\Lambda$

$q = \text{const} < 3$

$$H^2(z) = \Omega_m H_0^2 (1+z)^3 + (1-\Omega_m) H_0^2 (1+z)^q$$

$$w = \frac{q-3}{3} < 0$$

$$\frac{a}{a_0} = \left(\frac{\Omega_m}{1-\Omega_m}\right)^{\frac{1}{3-q}} \sinh^{\frac{2}{3-q}}\left((3-q)\sqrt{\frac{2\pi G}{q}}\,(\varphi - \varphi_0 + \varphi_1)\right)$$

$$V(\varphi) = \frac{3-\frac{q}{2}}{8\pi G} \cdot \frac{(1-\Omega_m)^{\frac{3}{3-q}}}{\Omega_m^{\frac{q}{3-q}}} H_0^2 \times$$

$$\times \frac{1}{\sinh^{\frac{2q}{3-q}}\left((3-q)\sqrt{\frac{2\pi G}{q}}\,(\varphi - \varphi_0 + \varphi_1)\right)}$$

$$\varphi_1 = \varphi_1(\Omega_m, q) = \frac{1}{3-q}\sqrt{\frac{q}{2\pi G}} \ln \frac{1+\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}}$$

Another interesting example:

$$\varepsilon_\Lambda = \varepsilon_v + \varepsilon_1 \left(\frac{a_0}{a}\right)^3$$

$$H^2 = H_0^2 \left(1 - \Omega_m - \Omega_1 + (\Omega_m + \Omega_1)\left(\frac{a_0}{a}\right)^3\right)$$

$$\varepsilon_v = \frac{3H_0^2}{8\pi G}(1 - \Omega_m - \Omega_1)$$

$$\varepsilon_1 = \frac{3H_0^2}{8\pi G}\Omega_1$$

$$V(\varphi) = \frac{3H_0^2}{8\pi G}\left(1 - \Omega_m - \Omega_1 + A \sinh^2(B(\varphi - \varphi_0 + \varphi_1))\right)$$

$$A = \frac{1}{2}\frac{\Omega_1(1-\Omega_m-\Omega_1)}{\Omega_m+\Omega_1} ; \quad B = \sqrt{\frac{6\pi G(\Omega_m+\Omega_1)}{\Omega_1}} ;$$

$$\varphi_1 = \sqrt{\frac{\Omega_1}{(\Omega_m+\Omega_1)\,24\pi G}}\ \ln\frac{1+\sqrt{\Omega_m+\Omega_1}}{1-\sqrt{\Omega_m+\Omega_1}}$$

$t \to \infty$: $a \to \infty$, $\varphi \to \varphi_0 - \varphi_1$

De Sitter with $m_\Lambda = \frac{3}{2} H_\infty$

$$\boxed{\Omega_1 \lesssim 0.05\,\Omega_m}$$

$$
\begin{tikzcd}
\mathcal{D}_L(z) \arrow[dr] & & \\
& H(z) \arrow[r, "\Omega_m"] & V(\varphi) \\
\left(\frac{\delta\rho}{\rho}\right)_{CDM}(z) \arrow[ur, "H_0"'] & &
\end{tikzcd}
$$

Reconstruction of $\delta(z)$ from $\mathcal{D}(z)$

$G_{eff}(z) = const$ assumed

Test if DE is physical or geometrical

$$E(z) = H_0\, \mathcal{D}_L(z)/(1+z) \Rightarrow z(E)$$

$$\delta(E(z)) = \delta(0) + \delta'(0)\int_0^E (1+z(E))\,dE$$

$$+ \frac{3}{2}\Omega_m \int_0^E (1+z(E_1))\,dE_1 \int_0^{E_1} \delta(E_2)\,dE_2$$

Given $\delta'(0)/\delta(0)$, $\delta(z)/\delta(0)$ can be found iteratively

No differentiation of data!

V. Sahni, A.S., IJMPD 15, 2105 (2006)
(astro-ph/0610026)

Practical reconstruction of $H(z)$, $w(z)$, etc. from $D_L(z)$

Explicit or implicit smoothing over some interval Δz is required!

1. Top-hat smoothing
2. Gaussian smoothing MNRAS 366, 1081 (2006) (A. Shafieloo et al., astro-ph/0505329)
3. The principal components method
4. Parametric fits (implicit smoothing!)

a) $$\frac{H^2(z)}{H_0^2} = A_0 + A_1(1+z) + A_2(1+z)^2 + \Omega_m(1+z)^3$$

$$A_0 + A_1 + A_2 + \Omega_m = 1$$

This fit does not exclude a possibility $\varepsilon_{DE} < 0$!

U. Alam et al. MNRAS 354, 275 (2004) [astroph/0311364]

U. Alam et al. JCAP 0604, 008 (2004) [astro-ph/0403687]

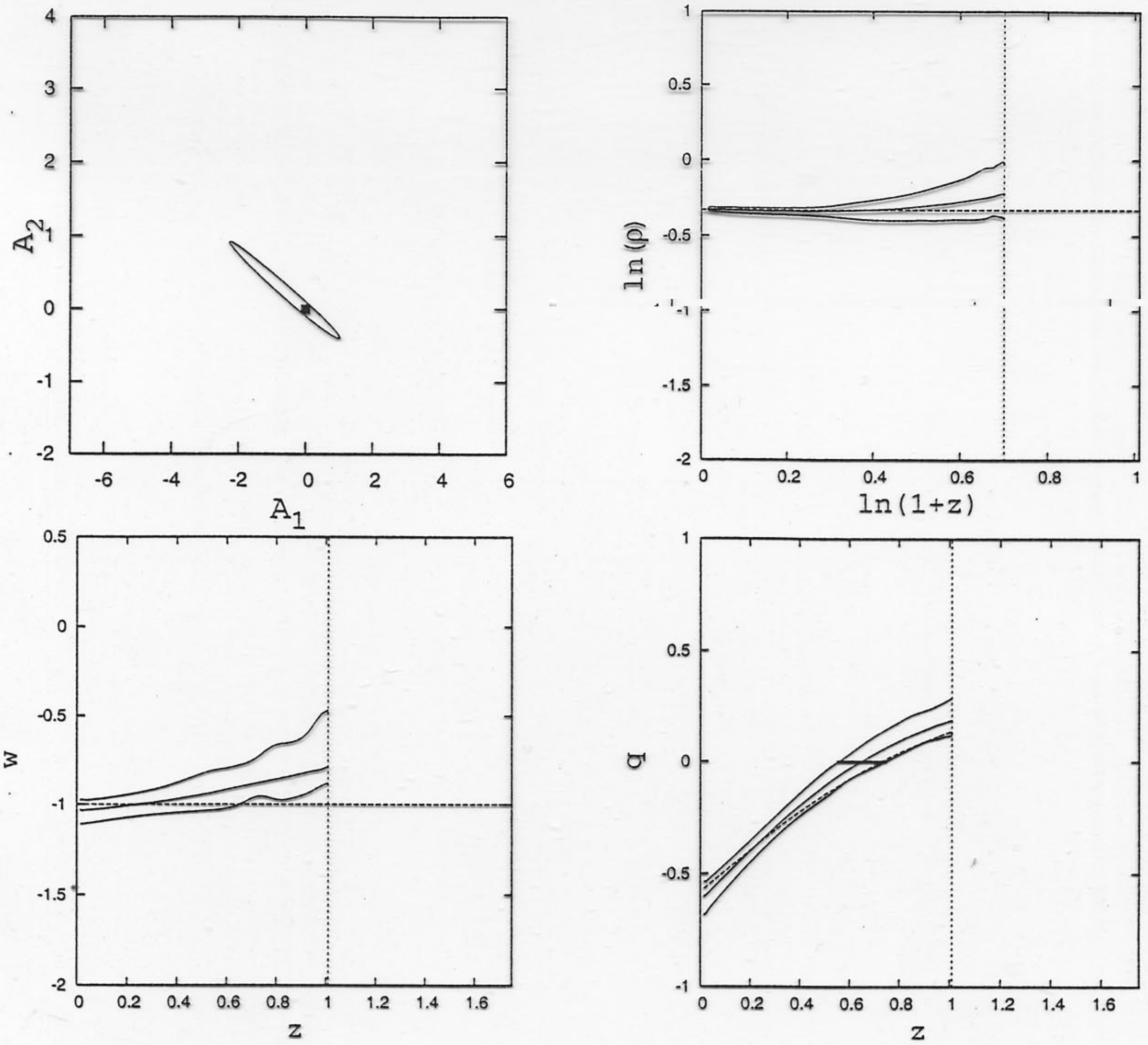
U. Alam et al. JCAP 0702, 011 (2007) [astro-ph/0612381]

b) The CPL fit

(Chevallier-Polarski-Linder)

$$w(z) = w_0 + w_1 \frac{z}{1+z}$$

U. Alam, V. Sahni, A. S., JCAP 0702, 011 (2007)
(astro-ph/0612381)



$w = \frac{p}{\rho}$

FIG. 7: 2σ confidence levels for the SNLS+CMB+BAO dataset using $\Omega_{0m} = 0.28 \pm 0.03$. The upper left hand panel shows the confidence levels in $A_1 - A_2$, with the black dot representing ΛCDM. The upper right hand panel shows the logarithmic 2σ variation of the DE density in terms of redshift. The dashed line represents ΛCDM. The lower left and right hand panels represent the variation of the equation of state and deceleration parameter respectively. The dashed lines in both panels represent ΛCDM. The thick solid line in the lower right hand panel shows the acceleration epoch, i.e. the redshift at which the universe started accelerating. Results are shown upto redshift $z = 1.01$

Fit:

$$\frac{H^2(z)}{H_0^2} = A_0 + A_1(1+z) + A_2(1+z)^2 + \Omega_m (1+z)^3$$

$$A_0 + A_1 + A_2 + \Omega_m = 1$$

E. Komatsu et al., arXiv: 0803.0547

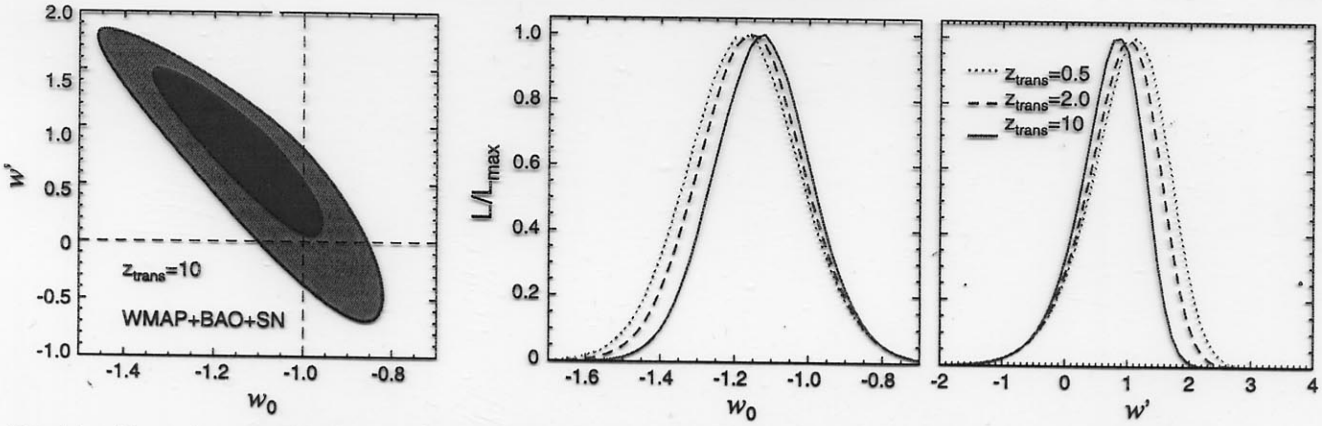


FIG. 14.— Constraint on models of time-dependent dark energy equation of state, $w(z)$ (Eq. [70]), derived from the *WMAP* distance priors (l_A, R, and z_*) combined with the BAO and SN distance data (§ 5.4.2). There are three parameters: w_0 is the value of w at the present epoch, $w_0 \equiv w(z=0)$, w' is the first derivative of w with respect to z at $z=0$, $w' \equiv dw/dz|_{z=0}$, and $z_{\rm trans}$ is the transition redshift above which $w(z)$ approaches to -1. Here, we assume flatness of the universe, $\Omega_k = 0$. (*Left*) Joint two-dimensional marginalized distribution of w_0 and w' for $z_{\rm trans} = 10$. The constraints are similar for the other values of $z_{\rm trans}$. The contours show the $\Delta\chi^2_{\rm total} = 2.30$ (68.3% CL) and $\Delta\chi^2_{\rm total} = 6.17$ (95.4% CL). (*Middle*) One-dimensional marginalized distribution of w_0 for $z_{\rm trans} = 0.5$ (dotted), 2 (dashed), and 10 (solid). (*Middle*) One-dimensional marginalized distribution of w' for $z_{\rm trans} = 0.5$ (dotted), 2 (dashed), and 10 (solid). The constraints are similar for all $z_{\rm trans}$. We do not find evidence for the evolution of dark energy.

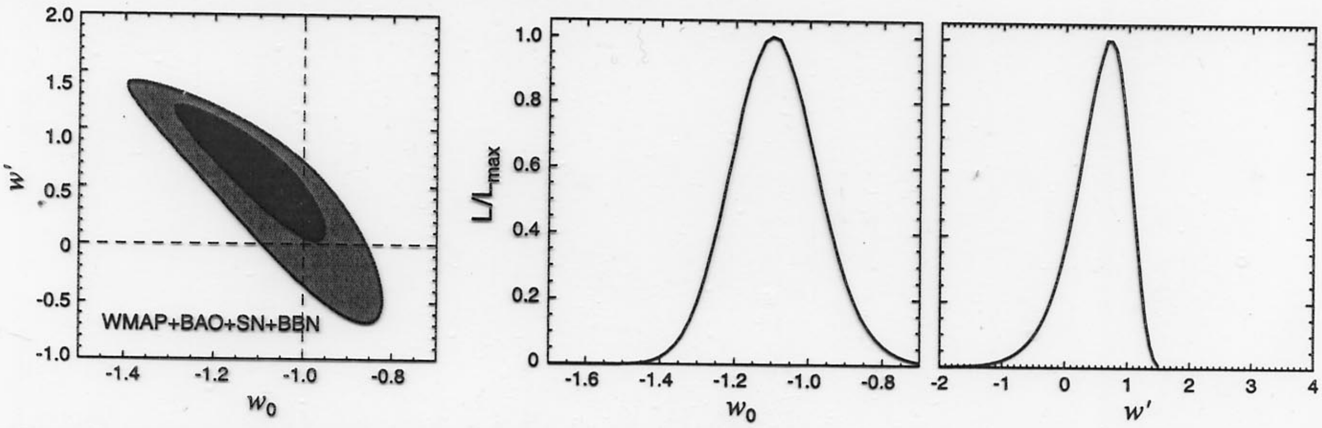


FIG. 15.— Constraint on the linear evolution model of dark energy equation of state, $w(z) = w_0 + w'z/(1+z)$, derived from the *WMAP* distance priors (l_A, R, and z_*) combined with the BAO and SN distance data as well as the Big Bang Nucleosynthesis (BBN) prior (Eq. [71]). Here, we assume flatness of the universe, $\Omega_k = 0$. (*Left*) Joint two-dimensional marginalized distribution of w_0 and w'. The contours show the $\Delta\chi^2_{\rm total} = 2.30$ (68.3% CL) and $\Delta\chi^2_{\rm total} = 6.17$ (95.4% CL). (*Middle*) One-dimensional marginalized distribution of w_0. (*Middle*) One-dimensional marginalized distribution of w'. We do not find evidence for the evolution of dark energy. Note that Linder (2003) defines w' as the derivative of w at $z=1$, whereas we define it as the derivative at $z=0$. They are related by $w'_{\rm linder} = 0.5w'_{WMAP}$.

The 95% limit on w_0 for $z_{\rm trans} = 10$ is $-0.38 < 1 + w_0 < 0.14$[46], whose upper limit is surprisingly close to the limit for a constant w model in a flat universe, $-0.097 < 1 + w < 0.142$ (95% CL). Therefore, we have a fairly robust upper limit on the present-day value of the equation of state, $1 + w_0 < 0.14$. (This statement is, however, true only for a flat universe.) On the other hand, the lower limit has weakened significantly – by a factor of about four. Our results are consistent with the previous work using the *WMAP* 3-year data (see Wang & Mukherjee 2007; Wright 2007, for recent work and references therein). We find that our upper limit on w_0 and lower limit on w' are better than the previous work by a factor of ~ 2.

Alternatively, one may take the linear form, $w(a) = w_0 + (1-a)w_a$, literally and extend it to an arbitrarily high redshift. This can result in an undesirable situation in which the dark energy is as important as the radiation density at the epoch of the Big Bang Nucleosynthesis (BBN); however, one can constrain such a scenario severely using the limit on the expansion rate from BBN (Steigman 2007). We follow Wright (2007) to adopt a Gaussian prior on

$$\sqrt{1 + \frac{\Omega_\Lambda(1+z_{\rm BBN})^{3[1+w_{\rm eff}(z_{\rm BBN})]}}{\Omega_m(1+z_{\rm BBN})^3 + \Omega_r(1+z_{\rm BBN})^4 + \Omega_k(1+z_{\rm BBN})^2}}$$
$$= 0.942 \pm 0.030, \qquad (71)$$

where we have kept Ω_m and Ω_k for definiteness, but they are entirely negligible compared to the radiation density at the redshift of BBN, $z_{\rm BBN} = 10^9$. Figure 15 shows the constraint on w_0 and w' for the linear evolution model derived from the *WMAP* distance priors, the BAO and SN data, and the BBN prior. The 95% limit on w_0 is

[46] The 68% intervals are $w_0 = -1.12 \pm 0.13$ and $w' = 0.70 \pm 0.53$ (WMAP+BAO+SN; $\Omega_k = 0$).

PRESENT STAGE OF DARK ENERGY RECONSTRUCTION

1. In the first approximation, DE is well described by a cosmological constant

$$w_{DE} \simeq -1$$

2. $w_{DE} = -1$ is inside 2σ error bars for all data

3. If $w_{DE} = \mathrm{const} \neq -1$, then

$$|w_{DE} + 1| \lesssim 0.1$$

E.g. W.J. Percival et al., arXiv: 0705.3323

$$w_{DE} = -1.004 \pm 0.089$$

WMAP5 + BAO + SN: $w_{DE} = -0.97 \pm 0.06$

4. No evidence for permanent phantom DE. No evidence for the Big Rip in future ($\Delta T > 50$ byl.y.)

$$a(t) \propto (t_1 - t)^{-p}$$

$$p > 0$$

However

5. SNe: small discrepancy between Gold and SNLS samples

BAO: comparison of $D_V(0.2)$ with $D_V(0.35)$ slightly favors $w_{DE} < -1$ for $z < 0.35$

6. If the assumption $w_{DE} = const$ is omitted, some place for 'temporary phantom' DE still exists for $z \lesssim 0.3$

But $\overline{w}_{DE}\,(0 < z \lesssim 0.5) \simeq -1$

WMAP5+BAO+SN: $-0.38 < 1 + w(0) < 0.14$ $\quad \dot{H}_0 < 0$

$-0.7 < w'(0) < 1.5$

7. Place for dynamical dark energy (especially, a geometric one) still exists!

Recent review on the reconstruction approach:

V. Sahni, A.A. Starobinsky, IJMPD 15, 2105 (2006) [astro-ph/0610026]

Interpretation of results

1. Conservative.

$\varepsilon_{DE} = const = \varepsilon_0$ agrees with all SN+CMB data (inside 2σ or better)

Especially good with Ly-α data from SDSS added and for SNLS data.

Models.

a) Casimir energy or vacuum polarization from additional compact or curved non-compact spatial dimensions

$$\varepsilon_{DE} = \frac{C}{R_d^4}$$

$D = 4 + d$
$d = 2$ flat compact
$d = 1$ AdS

$R_d < 5 \cdot 10^{-3}$ cm $\rightarrow$ $0 < C < 0.1$ (D.J. Kapner et al. hep-ph/0611184)

Deviation from the Newton law at $R \lesssim R_d$ – the most crucial test for these class of models

b) String theory de Sitter vacua – have appeared in fantastically large amount recently

Models of dynamical dark energy

Practical use of the remarkable similarity between primordial DE (supporting inflation) and present DE: the same models may be used for description of both inflation and present dark energy

Single inflation	Quintessence
$R+R^2$ model	$F(R)$ model
Extended inflation	Scalar-tensor DE
k-inflation	k-essence
Brane inflation	Brane DE
String inflation	String DE

The most critical problem:

"Graceful exit"	"Graceful entrance"

Model requirements

1. Stability of the Minkowski space-time with respect to perturbations with $\omega^2 \gg H_0^2$
   a) absence of tachyons,
   b) absence of ghosts
2. Solar System tests
3. Stability of the MD-stage

Example: $\mathcal{L} = f(R) \Rightarrow \begin{cases} \frac{df}{dR} > 0 \\ \frac{d^2f}{dR^2} \geq 0 \end{cases}$

4. MD- and RD-stages should be generic

Physical DE models

1. Quintessence = minimally coupled scalar field with some potential ("inflaton today")

$$\varepsilon = \frac{\dot{\varphi}^2}{2} + V\,, \quad p = \frac{\dot{\varphi}^2}{2} - V$$

No crossing of $w = -1$ line

If $V \propto \varphi^{-n} \Longrightarrow n < 1$

Slow-roll regime: $w \simeq -1 + \frac{\dot{\varphi}^2}{V} \simeq -1$

The Chaplygin gas model

$$p = -\frac{\rho_0^2}{\rho}$$

$$c_s^2 \equiv \frac{dp}{d\rho} = \frac{\rho_0^2}{\rho^2} > 0$$

$\rho > \rho_0$ for the "usual" model

$$\left(\begin{array}{c} \rho + p > 0 \\ c_s^2 < 1 \end{array} \right)$$

$$\rho = \sqrt{\rho_0^2 + C(1+z)^6}$$

Unifies DM and DE : can describe both the MD stage in the past and the transition to the Λ-dominated stage today

Equivalent field-theoretical models:

a) Quintessence with

$$V(\varphi) = \frac{\rho_0}{2}\left(\cosh(2\sqrt{6\pi G}\,\varphi) + \frac{1}{\cosh(2\sqrt{6\pi G}\,\varphi)}\right)$$

Equivalence for some solutions

(see V. Gorini et al., PRD 72, 103518 (2005); astro-ph/0504576)

b) k-essence $\mathcal{L} = -\rho_0\sqrt{1 - T_{,\mu}T^{,\mu}}$

Equivalence for all solutions with $T_{,\mu}T^{,\mu} > 0$

However, λ_J is too large!

$\lambda_J \propto c_s t \propto t^3$ at the MD stage

Perturbations stop growing for

$z \sim 3$ for the present comoving scale 100 Mpc

$z \sim 14$ —— " —— " —— " —— 1 Mpc

Wrong $P(k)$ today!

Too simple to describe dark energy

Generalized Chaplygin gas

$$p = -\rho_0 \left(\frac{\rho}{\rho_0}\right)^{-\alpha}$$

$\alpha > 0$

$$c_s^2 = \alpha \left(\frac{\rho}{\rho_0}\right)^{-\alpha-1} > 0$$

Equivalent k-essence model

$$\mathcal{L} = -\rho_0 \left[1 - \left(\frac{X}{\rho_0}\right)^{\frac{1+\alpha}{2\alpha}}\right]^{\frac{\alpha}{1+\alpha}} \qquad X = \frac{1}{2}\varphi_{,\mu}\varphi^{,\mu} < \rho_0$$

1. $0 < \alpha < 1$ – subluminal case

   Comparison with data: $\alpha \ll 1$

2. $\alpha > 1$ – superluminal case for $\frac{\rho}{\rho_0} < \alpha^{\frac{1}{1+\alpha}}$

V. Gorini et al., JCAP 0802(2008)016 (arXiv:0711.4242)

$\alpha \sim 3$ satisfies CMB and BAO data

The way to satisfy the causality condition

$c_s = 1$ for $X \to 0$ (at the characteristic)

for $X \ll \rho_1 \ll \rho_0$, change $\mathcal{L}$ to:

$$\mathcal{L}_1 = -\rho_0 + X\left(\frac{\rho_0}{\rho_1}\right)^{\frac{\alpha-1}{2\alpha}} \qquad c_s^2 = 1$$

$\mathcal{L}_1$ smoothly matches $\mathcal{L}_0$ for $X \sim \rho_1$

Corresponds to a change in the EOS for the generalized Chaplygin gas in the remote future

$\left(\text{for } \frac{\rho - \rho_0}{\rho_0} \sim \frac{\rho_1}{\rho_0} \ll 1\right)$

"Superluminality" is a transient effect disappearing for $t \to \infty$

DE models with "gravity leaking to higher dimensions"

The simplest model (Dvali et al., 2000)

Gravity in the $D=5$ bulk + induced gravity on the brane

$$H^2 = \left(\sqrt{\frac{8\pi G\rho}{3} + \frac{1}{4r_c^2}} + \frac{1}{2r_c}\right)^2$$

$$H_0 r_c = \frac{1}{1-\Omega_m} \simeq 1.4 \quad (1.25 \text{ for } \Omega_m = 0.2)$$

$\Omega_m = 0.3$

Sahni & Shtanov (2002, 2004) – generalization admitting $w_{DE} < -1$ and/or $\varepsilon_{DE} < 0$

$$a = a_0 \sinh^{2/3} \varphi$$

$$\frac{3t}{2r_c} = \varphi - \frac{e^{-2\varphi}}{2} + \frac{1}{2}$$

$$\Omega_m(t) = e^{-2\varphi}$$

$$q(t) = \frac{2\Omega_m - 1}{1+\Omega_m}$$

$$r(t) = 1 - \frac{9\Omega_m^2(1-\Omega_m)}{(1+\Omega_m)^3}$$

$\Omega_m = 0.3$: $\quad q_0 = -0.31, \; r_0 = 0.74$

This model is on verge to be falsified (using $\frac{\Delta T}{T}$ and $\frac{\delta\rho}{\rho}$)

Has a ghost (see, e.g., hep-th/0610282)

Gives a rather bad fit to the UNION SNe dataset (D. Rubin et al., arXiv:0807.1108)

What if recent phantom behaviour ($w<-1$) of dark energy will be confirmed by observations?

Ghost phantom models of dark energy are bad.

1. Quantum instability

2. At the classical level:

does not explain homogeneity and isotropy of the Universe

E.g.: for a given $\bar{H} = \frac{1}{3}\frac{d}{dt} \ln abc$, it is much more probable to have very different $\frac{\dot{a}}{a}, \frac{\dot{b}}{b}, \frac{\dot{c}}{c}$ compensated by the negative energy density of the ghost field.

Scalar-tensor models of dark energy do not have this problem

Reconstruction of dark energy in scalar-tensor gravity

B. Boisseau, G. Esposito-Farese, D. Polarski, A. S.
Phys. Rev. Lett. 85, 2236 (2000)

$\varepsilon_{DE} + p_{DE} < 0$ is permitted

$$\mathcal{L} = \frac{1}{2}\left(F(\varphi)R + Z(\varphi)\varphi_{,\mu}\varphi^{,\mu}\right) - V(\varphi) + \mathcal{L}_m$$

Includes $R + f(R)$ theory for $Z(\varphi) = 0$.

$Z(\varphi) = 1$ $\qquad$ $\omega^{-1}(\varphi) = F^{-1}\left(\frac{dF}{d\varphi}\right)^2$

Two independent observable cosmological functions are required for reconstruction of $F(\varphi)$ and $V(\varphi)$

$$D_L(z), \quad \delta(z)$$

$$\begin{array}{ccc} D_L(z) & & \delta(z) \\ \downarrow & & \downarrow \\ H(z) & \longrightarrow & F(z) \Longrightarrow V(z), \ \varphi(z) \end{array}$$

Background equations

$$3FH^2 = \rho_m + \frac{\dot{\varphi}^2}{2} + V - 3H\dot{F}$$

$$-2F\dot{H} = \rho_m + \dot{\varphi}^2 + \ddot{F} - H\dot{F} \qquad \rho_m \propto a^{-3}$$

Their consequence:

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} - 3(\dot{H} + 2H^2)\frac{dF}{d\varphi} = 0$$

In terms of redshift:

$$F'' + \left[(\ln H)' - \frac{4}{1+z}\right]F' + \left[\frac{6}{(1+z)^2} - \frac{2(\ln H)'}{1+z}\right]F$$

$$= \frac{2V}{(1+z)^2 H^2} + 3(1+z)\left(\frac{H_0}{H}\right)^2 F_0\, \Omega_{m,0}$$

$$\varphi'^2 = -F'' - \left[(\ln H)' + \frac{2}{1+z}\right]F' + \frac{2(\ln H)'}{1+z}F$$

$$- 3(1+z)\frac{H_0^2}{H^2}F_0\, \Omega_{m,0}$$

I. First step → as in GR

$$H(z) = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1}$$

II. Equation for sufficiently small-scale inhomogeneities

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff}\,\rho_m\,\delta = 0$$

where $\delta \equiv \left(\frac{\delta\rho}{\rho}\right)_{CDM+bar}$ at a fixed comoving scale $\lambda = a(t)/k$ and

$$\frac{k^2}{a^2} \gg \max\left(\frac{d^2V}{d\varphi^2},\ H^2 \cdot \max\left(1, \frac{d^2F}{d\varphi^2}\right)\right);$$

$$G_{eff} = \frac{1}{8\pi F} \cdot \frac{F + 2\left(\frac{dF}{d\varphi}\right)^2}{F + \frac{3}{2}\left(\frac{dF}{d\varphi}\right)^2}$$

From this, excluding $d\varphi$:

a second-order differential equation for $F(z)$.

Properties of scalar-tensor models of dark energy

R. Gannouji, D. Polarski, A. Ranquet, A. S.
JCAP 09, 016 (2006) [astro-ph/0606287]

1. Temporary phantom behaviour and crossing of the phantom boundary $w = -1$ are possible for an open set of $F(\varphi)$ and non-zero and non-constant $V(\varphi)$.
   "Curvature induced phantomness"
2. In the absence of dust-like matter ($\Omega_m = 0$), power-law solutions leading to the Big Rip singularity in future and to $w < -1$ exist if

$$F = \alpha \varphi^2 , \quad \varphi \to \infty$$

(Barrow & Maeda 1990)

$$V = V_0 |\varphi|^n , \quad 2 < n < 4$$

Then $a(t) \propto (t_0 - t)^q$, $\quad q = \dfrac{2\left(n + 2 + \frac{1}{\alpha}\right)}{(n-2)(n-4)} < 0$

$\varphi(t) \propto (t_0 - t)^r$, $\quad r = \dfrac{2}{2-n} < 0$

However, for these solutions $|w+1| \le \frac{\alpha}{3} \sim \frac{1}{\omega_{BD}}$ and very small.

Present observational bounds:

$$\gamma_{PN} - 1 = (2.1 \pm 2.3) \cdot 10^{-5}$$ Bertotti et al., 2003 → Cassini mission

$$\beta_{PN} - 1 = (0 \pm 1) \cdot 10^{-4}$$ Pitjeva, 2005 → ephemerides of planets

$$\frac{\dot{G}_{eff,0}}{G_{eff,0}} = (-0.2 \pm 0.5) \cdot 10^{-13} y^{-1}$$

$$\beta_{PN} - 1 = (1.2 \pm 1.1) \cdot 10^{-4}$$ Williams et al., 2005 → lunar laser ranging

$$\omega_{BD,0} \equiv \left(\frac{F}{\left(\frac{dF}{d\varphi}\right)^2} \right)_0 > 4 \cdot 10^4$$

3. Small z expansion.

$$\frac{F(z)}{F_0} = 1 + F_1 z + F_2 z^2 + \ldots$$

$$\frac{V(z)}{3 F_0 H_0^2} = \Omega_{V,0} + u_1 z + u_2 z^2 + \ldots$$

$$\frac{H^2(z)}{H_0^2} = 1 + h_1 z + h_2 z^2 + \ldots$$

$$F_0^{-1/2} \varphi'(z) = \varphi_0' + \varphi_1' z + \varphi_2' z^2 + \ldots$$

$$w_{DE}(z) = w_0 + w_1 z + w_2 z^2 + \ldots$$

$$|F_1| < 10^{-2}$$

$$\varphi_0'^2 = 6(1 - \Omega_{m,0} - \Omega_{V,0} - F_1) \geq 0$$

What is required to get significant phantomness ($|w+1| \gg \frac{1}{\omega_{BD,0}}$) ?

$$F_2 < 0\,, \quad |F_2| \sim 1 \gg |F_1| \qquad (|F_1| < 10^{-2})$$

$$|F_2| > 3\,(\underbrace{\Omega_{DE,0}}_{\simeq 1-\Omega_{m,0}} - \Omega_{V,0}) > 0$$

$$w_0 + 1 = \frac{2F_2 + 6(\Omega_{DE,0} - \Omega_{V,0})}{3\Omega_{DE,0}} < 0$$

F_1 can be negative, too

4. Connection with post-Newtonian parameters in the significantly phantom case.

$$\gamma_{PN} - 1 = -\frac{F_1^2}{6(\Omega_{DE,0} - \Omega_{V,0})} < 0$$

$$\beta_{PN} - 1 = -\frac{F_1^2 F_2}{72(\Omega_{DE,0} - \Omega_{V,0})} > 0$$

$$-4 < \frac{\gamma_{PN} - 1}{\beta_{PN} - 1} = \frac{12(\Omega_{DE,0} - \Omega_{V,0})}{F_2} < 0$$

However, $|\gamma_{PN} - 1|$ and $|\beta_{PN} - 1|$ may be much smaller than $|1+w|$ if F_1 is very small

$$\frac{\dot{G}_{eff,0}}{G_{eff,0}} = H_0 F_1 \left(1 - \frac{F_2}{3(\Omega_{DE,0} - \Omega_{V,0})}\right)$$

Positive detection of $\gamma_{PN} < 1$, $\beta_{PN} > 1$ may be a strong argument for significant phantomness of present DE.
Negative detection tells nothing.

5. Correct asymptotic behaviour for large z ($\varphi'^2 \geq 0$, $w_{DE} \leq 0$) requires non-zero and non-constant $V(\varphi)$

E.g. $F(\varphi) \to F_\infty < F_0$

$$V(\varphi) \propto \exp\left(\sqrt{\frac{3}{2F_0 \Omega_{u,\infty}}}\,\varphi\right)$$

$$z, \varphi \to \infty$$

6. In the stable case $F > 0$, $\omega_{BD} > -\frac{3}{2}$, no possibility to construct a stable wormhole (even with an electromagnetic field)
(K. A. Bronnikov & A. S., JETP Lett. 85, 1 (2007) [gr-qc/0612032])
7. In the absence of ghosts and positive spatial curvature, and for $u \geq 0$, bounce in a FRW model ($H = 0$, $\dot{H} > 0$) is <u>not</u> possible

Geometrical $f(R)$ model of DE

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R) \qquad R \equiv R_\mu^\mu$$

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = -8\pi G\left(T_{\mu(m)}^\nu + T_{\mu(DE)}^\nu\right)$$

$$8\pi G\, T_{\mu(DE)}^\nu \equiv F'(R) R_\mu^\nu - \frac{1}{2}F(R)\delta_\mu^\nu$$

$$+ \left(\nabla_\mu \nabla^\nu - \delta_\mu^\nu \nabla_\rho \nabla^\rho\right) F'(R)$$

Particle content: graviton + massive scalar particle $\left(M^2 = \frac{1}{3f''(R)}\right)$ (dubbed "scalaron" in A.S., 1980)

Stability conditions:

1. $f' > 0$ graviton is not a ghost
2. $f'' > 0$ scalaron is not a tachyon

imposed for $R \geq R_{now}$ at least (i.e. during the whole evolution of the Universe)

Violation of these conditions is undesirable from the classical point of view, too !

$f'(R_0) = 0$ - instant loss of homogeneity and isotropy

$f''(R_0) = 0$ - weak singularity

$$R(t) = R_0 + O(\sqrt{t})$$
$$a(t) = a_0 + a_1 t + a_2 t^2 + O(t^{5/2})$$

③ Existense of the Newtonian regime

$$(\Delta \varphi = 4\pi G \rho)$$

$$|F| \ll R, \ |F'(R)| \ll 1, \ R|F''(R)| \ll 1$$

for $R_{now} \lll R$ (at up to some very large R)

$\longleftrightarrow$

De Sitter regime

$$Rf' = 2f$$

Stable if

$$f'(R_1) > R_1 f''(R_1)$$

$\longleftrightarrow$

Equivalent to $\omega_{BD} = 0$ scalar-tensor gravity

Use for inflation

$$f(R) = R + \frac{R^2}{6M^2}$$ (+ small non-local terms)

AS, 1980

Internally self-consistent inflationary model with slow-roll decay, a graceful exit to the subsequent RD FRW stage (through an indermediate matter-dominated stage) and sufficiently effective reheating

$\tau \sim M_{Pl}^2/M^3$ $\quad N \sim 50$

Remains viable

$$M = 3.0 \times 10^{-6} (N/50)^{-1} M_{Pl}$$

$$n_s = 1 - \frac{2}{N} = 0.96 \qquad \text{for } N = 50$$

$$r = \frac{12}{N^2} = 4.8 \times 10^{-3} (N/50)^{-2}$$

Eksp. : $\overline{n_s} = 0.96 \pm 0.014$, $r < 0.20$

Use for DE

$F(R) \propto R^{-n}$ for $R \to 0$

Does not work for many reasons

Viable model – regular at $R=0$

$$f(R) = R + \lambda R_0 \left(\frac{1}{\left(1+\frac{R^2}{R_0^2}\right)^n} - 1 \right)$$

AS, JETP Lett. 86, 157 (2007)
arXiv: 0706.2041 [astro-ph]

or even

$$f(R) = R - \lambda R_0 \tanh^2 \frac{R}{R_0}$$

$f(0) = 0$ – 'disappearing' cosmological constant in flat space-time

Induced Λ at high curvatures:

$$\Lambda_\infty \equiv -\frac{1}{2} F(\infty) = \frac{\lambda R_0}{2}$$

Observational restrictions

1. Cosmology

Anomalous growth of non-relativistic matter perturbations in the regime

$k \gg M(R)\, a$

$G_{eff} = 4G/3f'(R) \simeq \frac{4G}{3}$

$\left(\frac{\delta\rho}{\rho}\right)_m \propto t^{\frac{\sqrt{33}-1}{6}}$ (instead of $\propto t^{2/3}$)

Results in the apparent mismatch

$$\Delta n_s = n_s^{(gal)} - n_s^{(CMB)} = \frac{\sqrt{33}-5}{2(3n+2)}$$

$\Delta n_s < 0.05 \longrightarrow n \geqslant 2$

$n = 2$ $\left(F(R) \propto R^{-4}\right) \Rightarrow$ increase of σ_8 from 0.80 to 0.95

2. Laboratory and Solar system tests

$M(R)\, L \gg 1$ with $R = 8\pi G T_m = 8\pi G \rho_m$

otherwise, $\gamma_{PN} = \frac{1}{2}$ and the 'fifth' force appears

$M(R(\rho_m)) \propto \rho_m^{n+1}$ $\quad (R = 8\pi G \rho_m)$

$n \geqslant 2$ is sufficient for all tests

Structure of corrections at the matter-dominated and earlier stages

$$R = R^{(0)} + \delta R_{ind} + \delta R_{osc}$$

$$R^{(0)} = 8\pi G T_m \propto a^{-3}$$

$$\delta R_{ind} = \left(R F'(R) - 2F(R) - 3 \nabla_\mu \nabla^\mu F'(R) \right)_{R=R^{(0)}}$$

$R \gg R_0$: $\delta R_{ind} \simeq const = -2F(\infty) = 4\Lambda(\infty)$

No Dolgov-Kawasaki instability

$$\delta R_{osc} \propto \begin{cases} t^{-3n-4} \sin(const \cdot t^{-2n-1}) & \text{MD} \\ t^{-\frac{3n}{4}-3} \sin(const \cdot t^{-(3n+1)/2}) & \text{RD} \end{cases}$$

$\frac{\delta a}{a}$ is small but $\frac{\delta R_{osc}}{R^{(0)}}$ diverges for $t \to 0$

δR_{osc} should be very small just from beginning – problem for those $F(R)$ models which do not let R become negative (due to the crossing of the $F''(R) = 0$ point)

"Scalaron overproduction" problem

"Big Boost" SINGULARITY WITH $R \to \infty$ AND ITS ELIMINATION

If $F(R) \to 0$ at $R \to \infty$, then a new generic "Big Boost" singularity can arise:

$F(R) \propto R^{-2n}$; $F''(\infty) = 0$

$$a = a_0 + a_1 (t - t_0) + a_2 |t - t_0|^k, \quad 1 < k = \frac{2n+1}{n+1} < 2$$

$$R \propto |t - t_0|^{k-2} > 0$$

Elimination:

add $\frac{R^2}{6M^2}$ to $F(R)$ $\qquad M^2(\infty) = M$

Additional advantages:

1. No unlimited growth of $M(R)$
2. A toy "UV-completion" – further radiative corrections are logarithmic only
3. A possibility to unify inflation and present dark energy in one $F(R)$ model if $M = 3 \cdot 10^{-6} M_{Pl}$

$\tau_{dec} \sim \frac{M_{Pl}^2}{M^3} \longrightarrow$ $M > 10^4$ GeV for scalarons to decay before BBN

CONCLUSIONS FOR $f(R)$ MODELS

1. With a regular $f(R)$ satisfying

$f'(R) > 0,\ f''(R) > 0$ for all R

$|f - R| \ll R,\ |f' - 1| \ll 1,\ R|f''| \ll 1$ for $R_0 \ll R \ll M^2$ with $M \gtrsim 10^4$ GeV

$f(R) \propto R^2$ for $R \to \infty$,

it is possible to construct viable models of DE, satisfying all existing cosmological, Solar system and laboratory data, and distinguishable from ΛCDM

2. Further unification of primordial DE (producing inflation) and present DE is possible for the specific choice of M: $M \approx 3 \cdot 10^{-6} M_{Pl}$

3. The most critical test of these DE models: anomalous growth of scalar perturbations at recent time ($z \sim 1-3$ for $L = 8h^{-1}$ Mpc)

CONCLUSIONS

1. Deviation of dynamical DE from an exact cosmological constant is $\lesssim 10\%$, but still may exist

2. The simplest DE model which can accomodate its possible recent phantom behaviour and crossing of the "phantom boundary" $w_{DE} = -1$ is based on scalar-tensor gravity and does not have ghosts or instabilities

3. Viable models in $f(R)$ gravity, though more restricted, are possible, too

4. However close the present DE may be to Λ, simply by analogy with primordial DE, one should not expect it to be absolutely stable and eternal