Azimuthal Correlation between the $(\vec{p}_l, \vec{p}_{X_b})$ and $(\vec{p}_l, \vec{P}_t)$ plains in the semileptonic rest frame decay of a polarized top quark: An $O(\alpha_s)$ effect

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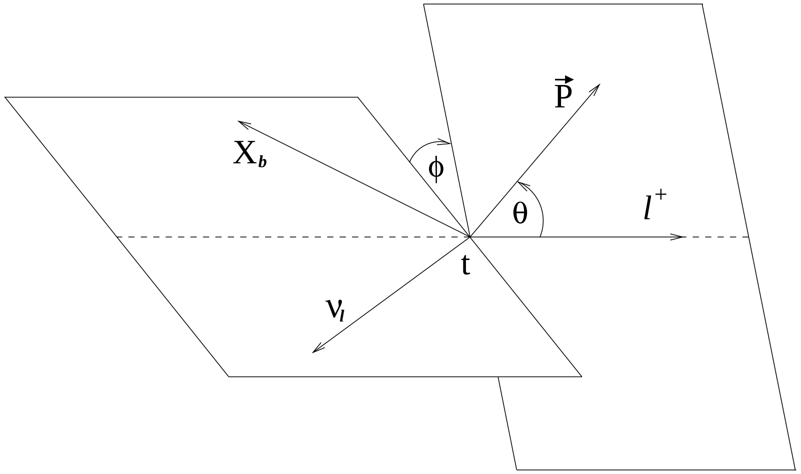
Outline:

- Introduction to the angular rate structure for $t(\uparrow) \to b + W^+(\ell^+\nu)$
- LO Born term result
- NLO QCD corrections:
  - Virtual correction
  - Real correction
- Non-SM right chiral quark current
- Conclusions

1. The general angular decay distribution

The decay: $t(\uparrow) \to b + W^+(\ell^+\nu)$ is described by

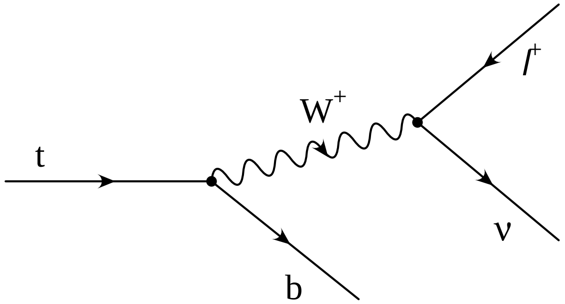
$$\frac{d\Gamma}{dx d\cos\theta d\phi} = \frac{1}{4\pi}\Big[\frac{d\Gamma_A}{dx} + P(\frac{d\Gamma_B}{dx}\cos\theta + \frac{d\Gamma_C}{dx}\sin\theta\cos\phi)\Big]$$



$$s_t^\mu = (0; \sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$
$$\vec{P} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

where x is the scaled energy of lepton in top rest frame: $x = \frac{2E_l}{m_t}$.

2. The LO Born term results



$$\text{The matrix element :} \quad M \to i\frac{G_F V_{tb}}{\sqrt{2}} H_\mu L^\mu$$

with leptonic and hadronic currents

$$H_\mu = \bar{u}_b \gamma_\mu (1 - \gamma_5) u_t, \qquad \text{hadronic current,}$$

$$L_\mu = \bar{u}_\nu \gamma_\mu (1 - \gamma_5) v_\ell, \qquad \text{leptonic current.}$$

The decay rate

$$\overline{|M|^2} = \sum_{s_b, s_\ell, s_\nu} |M|^2 = \sum_{s_b, s_\ell, s_\nu} M M^\dagger \to \frac{G_F^2 |V_{tb}|^2}{2} H_{\mu\nu} L^{\mu\nu}$$

with hadron and lepton tensors

$$H_{\mu\nu} = \sum_{s_b} H_\mu H_\nu^\dagger, \qquad L_{\mu\nu} = \sum_{s_\ell, s_\nu} L_\mu L_{\nu^\dagger}.$$

⋆ narrow resonance approximation

⋆ zero bottom quark mass

The LO differential rate is

$$\frac{d\Gamma_A}{dx} = \frac{d\Gamma_B}{dx} = \Gamma_F \, 2\pi \frac{m_W}{\Gamma_W} 6x(1-x)y,$$

$$\frac{d\Gamma_C}{dx} = 0\,.$$

The LO integrated rate $(y \leq x \leq 1)$ is

$$\Gamma_A = \Gamma_B = \Gamma_F \, 2\pi \frac{m_W}{\Gamma_W} y(1-y)^2(1+2y),$$

$$\Gamma_C = 0\,.$$

where $\Gamma_F = \frac{G_F^2 m_t^5}{192\pi^3} |V_{tb}|^2, y = \frac{m_W^2}{m_t^2}$.

- $\Gamma_C = 0$ is a direct result of (V-A)(V-A) current–current structure

$$\begin{aligned} M &= \bar{u}_b \gamma_\mu \left(1-\gamma_5\right) u_t \bar{u}_\nu \gamma^\mu \left(1-\gamma_5\right) v_\ell \\ &\downarrow \quad \text{Fierz transformation} \\ &= 2\bar{u}_b \left(1+\gamma_5\right) v_\nu \bar{u}_\ell \left(1-\gamma_5\right) u_t \end{aligned}$$

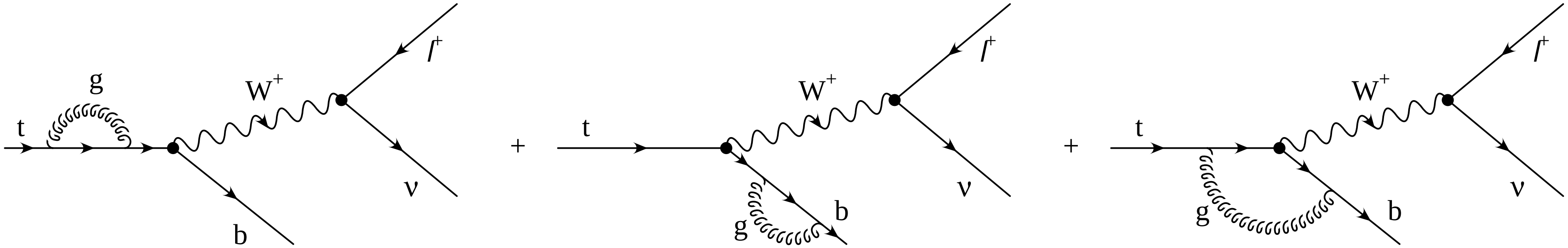
$$\sum_{spins} |M|^2 = (p_b \cdot p_\nu)\ 4E_l m_t (1+\cos\theta) \Rightarrow \Gamma_A = \Gamma_B\ ,\ \Gamma_C = 0$$

- $\Gamma_C = 0$ also follows from $\Gamma_A = \Gamma_B$, if Γ is to be positive definite

$$\Gamma \sim \Gamma_A (1 + \cos\theta + \frac{\Gamma_C}{\Gamma_A} \sin\theta \cos\phi)$$

- It is interesting to find out how much this result is changed by *radiative corrections*.

3. The Virtual Correction



The renormalized hadron current of $O(\alpha_s)$ can be written as:

$$\boxed{H_\mu^{(1)} = \bar{u}_b (J_\mu^V - J_\mu^A) u_t,}$$

The covariant expansion of J_μ^V and J_μ^A is:

$$\begin{aligned} J_\mu^V &= \gamma_\mu F_1^V + p_{t,\mu} F_2^V + p_{b,\mu} F_3^V, \\ J_\mu^A &= (\gamma_\mu F_1^A + p_{t,\mu} F_2^A + p_{b,\mu} F_3^A)\gamma_5. \end{aligned}$$

At the one-loop level the form factors are given by

$$
\begin{aligned}
F_1^V = F_1^A &= 1-\frac{\alpha_s(q^2)}{4\pi}C_F\Bigg(4+\frac{1}{y}\ln(1-y)+\ln\left(\frac{y}{1-y}\frac{\Lambda^4}{(1-y)^2}\right)+\\
&\qquad 2\ln\left(\frac{\Lambda^2}{y}\frac{1}{1-y}\right)\ln\left(\frac{y}{1-y}\right)+2Li(y)\Bigg),\\
F_2^V = -F_2^A &= \frac{1}{m_t}\frac{\alpha_s(q^2)}{4\pi}C_F\,\frac{2}{y}\Bigg(+1+\frac{1-y}{y}\ln(1-y)\Bigg),\\
F_3^V = -F_3^A &= \frac{1}{m_t}\frac{\alpha_s(q^2)}{4\pi}C_F\,\frac{2}{y}\Bigg(-1+\frac{2\,y-1}{y}\ln(1-y)\Bigg),
\end{aligned}
$$

where a gluon mass regulator $\Lambda = m_g/m_t$ is used to regularize the gluon IR singularity.

The hadron tensor for the virtual correction is

$$
\begin{aligned}
H_{\mu\nu} \quad &= \sum_{s_b} H_\mu H_\nu^\dagger = \sum_{s_b} \left(H_\mu^{(0)} + H_\mu^{(1)}\right)\left(H_\nu^{(0)} + H_\nu^{(1)}\right)^\dagger \\
&= \sum_{s_b} H_\mu^{(0)} H_\nu^{(0)\dagger} + \sum_{s_b} \left(H_\mu^{(0)} H_\nu^{(1)\dagger} + H_\mu^{(1)} H_\nu^{(0)\dagger}\right) + \mathcal{O}(\alpha_s^2) \\
&= H_{\mu\nu}^{(0)} + H_{\mu\nu}^{(1)} + \mathcal{O}(\alpha_s^2)
\end{aligned}
$$

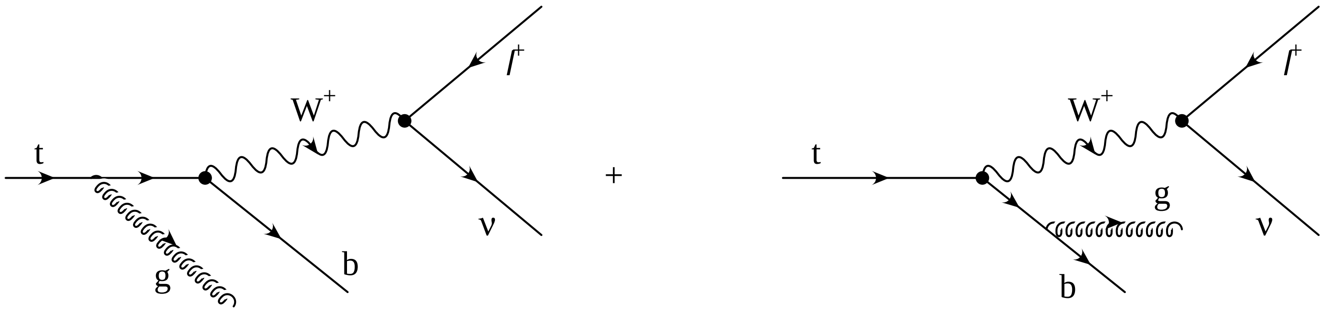
The $O(\alpha_s)$ virtual correction to the decay rate

$$
\overline{|M^{(1)}|^2} = \sum_{s_b, s_\ell, s_\nu} |M^{(1)}|^2 \to \frac{G_F^2 |V_{tb}|^2}{2} H_{\mu\nu}^{(1)} L^{\mu\nu}
$$

- **No IR-singularity** because F_1^V comes with the form

$$
F_1^V \bar{u}_b \gamma_\mu (1 - \gamma_5) u_t .
$$

4. The Real Correction



$$H_\mu = \bar{u}_b \left(\frac{\gamma_\mu \not{k} \gamma_\alpha - 2p_{t\alpha}\gamma_\mu}{2p_t \cdot k} + \frac{\gamma_\alpha \not{k} \gamma_\mu + 2p_{b\alpha}\gamma_\mu}{2p_b \cdot k} \right) (1 - \gamma_5)\, \epsilon_\lambda^{\alpha *} u_t .$$

The hadron tensor for the real correction is

$$
\begin{aligned}
\mathcal{H}^{\mu\nu} \quad = \quad & -4\pi\alpha_s\, C_F \, \frac{8}{(k\cdot p_t)(k\cdot p_b)} \Bigg\{ - \frac{k\cdot p_t}{k\cdot p_b} \Big[\\
& i\Big(\epsilon^{\alpha\beta\mu\nu}\,(p_b-k)\cdot\bar{p}_t - \epsilon^{\alpha\beta\gamma\nu}(p_b-k)^\mu\,\bar{p}_{t,\gamma} + \epsilon^{\alpha\beta\gamma\mu}(p_b-k)^\nu\,\bar{p}_{t,\gamma}\Big) k_\alpha\, p_{b,\beta}\Big] + \\
& \frac{k\cdot p_b}{k\cdot p_t}\Big[(\bar{p}_t\cdot p_t)\Big(k^\mu\, p_b^\nu + k^\nu\, p_b^\mu - k\cdot p_b\, g^{\mu\nu} - i\,\epsilon^{\alpha\beta\mu\nu} k_\alpha\, p_{b,\beta}\Big) \\
& - (\bar{p}_t\cdot k)\Big((p_t-k)^\mu\, p_b^\nu + (p_t-k)^\nu\, p_b^\mu - (p_t-k)\cdot p_b\, g^{\mu\nu} - i\,\epsilon^{\alpha\beta\mu\nu}(p_t-k)_\alpha\, p_{b,\beta}\Big)\Big] \\
& - (\bar{p}_t\cdot p_b)\Big(k^\mu\, p_b^\nu + k^\nu\, p_b^\mu - k\cdot p_b\, g^{\mu\nu} - i\,\epsilon^{\alpha\beta\mu\nu} k_\alpha\, p_{b,\beta}\Big) + (p_t\cdot p_b)\Big(k^\mu\,\bar{p}_t^\nu + k^\nu\,\bar{p}_t^\mu - k\cdot\bar{p}_t\, g^{\mu\nu}\Big) \\
& - (k\cdot p_b)\Big(p_t^\mu\,\bar{p}_t^\nu + p_t^\nu\,\bar{p}_t^\mu - p_t\cdot\bar{p}_t\, g^{\mu\nu}\Big) + (k\cdot p_t)\Big((p_b+k)^\mu\,\bar{p}_t^\nu + (p_b+k)^\nu\,\bar{p}_t^\mu - (p_b+k)\cdot\bar{p}_t\, g^{\mu\nu}\Big) \\
& + (k\cdot\bar{p}_t)\Big(2p_b^\mu\, p_b^\nu - p_b\cdot p_b g^{\mu\nu}\Big) + i\Big(\epsilon^{\alpha\beta\mu\nu}\,(k\cdot\bar{p}_t) + \epsilon^{\alpha\beta\gamma\mu}\, k^\nu\bar{p}_{t,\gamma} - \epsilon^{\alpha\beta\gamma\nu}\, k^\mu\bar{p}_{t,\gamma}\Big) p_{b,\alpha}\, p_{t,\beta} \\
& + i\Big(\epsilon^{\alpha\beta\mu\nu}\,(p_t\cdot\bar{p}_t) + \epsilon^{\alpha\beta\gamma\mu}\, p_t^\nu\bar{p}_{t,\gamma} - \epsilon^{\alpha\beta\gamma\nu}\, p_t^\mu\bar{p}_{t,\gamma}\Big) k_\alpha\, p_{b,\beta} \Bigg\} + \mathbf{B}^{\mu\nu}\cdot\Delta_{\mathbf{SGF}}
\end{aligned}
$$

where

$$\bar{p}_t^\mu = p_t^\mu - m_t\, \boxed{s_t^\mu}$$

$$k \rightarrow \text{gluon momentum}$$

$$\Delta_{SGF} \rightarrow \textit{IR singular part}$$

$$B^{\mu\nu} = \text{the square of } \bar{u}_b\gamma_\mu\,(1-\gamma_5)\,u_t \Rightarrow \textbf{No IR – singularity}\,.$$

For the four-body phase space we used following Dalitz plot:

$$\begin{aligned} dR_4\,(p_t; p_b, \ell, \nu, k) &= dP^2 dR_3\,(p_t; P, \ell, \nu)\, dR_2\,(P; p_b, k) \\ &= m_t^2\, \boxed{\mathrm{dz}}\, dR_2\,(P; p_b, k)\, dR_3\,(p_t; P, \ell, \nu) \end{aligned}$$

where z is the scaled invariant mass squared of the b quark-gluon system: $z = \frac{P^2}{m_t^2} = \frac{(p_b+k)^2}{m_t^2}$.

The order α_s differential rates are

$$\frac{d\Gamma_i^{(1)}}{dxdz} = \Gamma_F 2\pi \frac{M_W}{\Gamma_W} C_F \frac{\alpha_s}{2\pi} 6y^2 \left[M_4^i(x,z) + M_3^i(x)\delta(z) \right], \ (i = A, B, C)$$

where $M_4^i(x,z)$ and $M_3^i(x)$ are the virtual and real corrections respectively.

For $i = A, B$ we have found agreement with Kühn, Jeźabek, Czarnecki, and the $i = C$ result is new.

$$\begin{aligned} M_4^C(x,z) &= \sqrt{y(1-x)(x-y) - xyz} \left(\frac{y}{x\lambda^3} j_1 + \frac{1}{\lambda^3} j_2 + \frac{x}{\lambda^3} j_3 \right. \\ &\quad \left. + 4Y_P \left[\frac{6yz}{x\lambda^{7/2}} j_4 + \frac{1}{\lambda^{7/2}} j_5 + \frac{x}{\lambda^{7/2}} j_6 \right] \right) \\ M_3^C(x) &= -\sqrt{y(1-x)(x-y)} \left(\frac{1-x}{y} \right) \ln(1-y), \end{aligned}$$

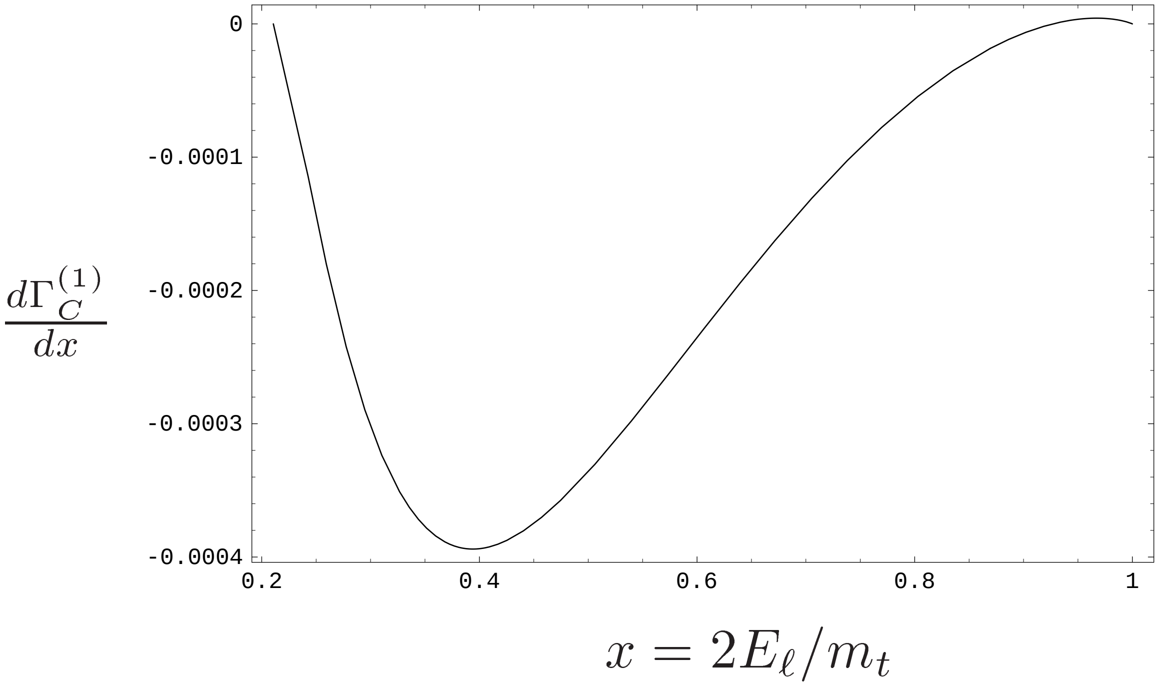
with $\lambda = 1 + y^2 + z^2 - 2(y + z + yz), Y_P = \frac{1}{2} \log \frac{1-y+z+\sqrt{\lambda}}{1-y+z-\sqrt{\lambda}}$ and j's are lengthy but linear functions of y and z.

- Difficulties in obtaining closed form for $\frac{d\Gamma_C}{dx}$

We could not obtain $\frac{d\Gamma}{dx}$ in closed form, because we have not been able to do following form of integrations *analytically*

☹ $$\int dz \frac{z^n \sqrt{y(1-x)(x-y) - xyz}}{\sqrt[m]{1+y^2+z^2-2(y+z+yz)}} \log \frac{1-y+z+\sqrt{1+y^2+z^2-2(y+z+yz)}}{1-y+z-\sqrt{1+y^2+z^2-2(y+z+yz)}}.$$

- Of course the numerical integration is possible and a plot is given:



However the closed form for $\Gamma_C^{(1)}$ is possible by changing the order of the integration

$$\Gamma_C = \int dx \int dz \frac{d\Gamma_C}{dxdz} = \int dz \int dx \frac{d\Gamma_C}{dxdz}.$$

Finally the closed form NLO integrated azimuthal correlation function is

☺
$$\begin{aligned}\Gamma_C^{(1)} \quad = \quad & \Gamma_F \, 2\pi \frac{M_W}{\Gamma_W} C_F \left(-\frac{\alpha_s}{2\pi}\right) \frac{\pi}{8} y^2 \Big[2\sqrt{y}(-4 - 3y + 3y^2)(2Li_2(\sqrt{y}) - Li_2(y)) \\ & + 2(1-y)(8 - 7\sqrt{y} + 8y - 5y^{3/2}) \ln(1+\sqrt{y}) - \frac{(1-y)^3}{\sqrt{y}} \ln(1-y) \\ & + \frac{1}{3}\sqrt{y}(6(1-\sqrt{y})^2(1-\sqrt{y}-2y) + \pi^2(4+3y-3y^2)) \Big].\end{aligned}$$

• Numerical results

Collecting the LO and NLO results will give

$$
\begin{aligned}
\frac{d\Gamma}{d\cos\theta\, d\phi} &= \frac{1}{4\pi}\Big[\frac{d\Gamma_A}{dx} + P(\frac{d\Gamma_B}{dx}\cos\theta + \frac{d\Gamma_C}{dx}\sin\theta\cos\phi)\Big] \\
&= \frac{\Gamma_A^{(0)}}{4\pi}\left[\left(1+\frac{\Gamma_A^{(1)}}{\Gamma_A^{(0)}}\right) + \left(1+\frac{\Gamma_B^{(1)}}{\Gamma_A^{(0)}}\right)P\cos\theta + \frac{\Gamma_C^{(1)}}{\Gamma_A^{(0)}}P\sin\theta\cos\phi\right] \\
&= \frac{\Gamma_A^{(0)}}{4\pi}\Big[\big(1-\underline{8{,}56}\,\%\big) + \big(1-\underline{8{,}72}\,\%\big)\,P\cos\theta - \underline{0{,}24}\,\%\,P\sin\theta\cos\phi\Big].
\end{aligned}
$$

• Roughly 9% corrections to the *unpolarized* and *polarized* rate.

• 0,24% correction to the *azimuthal* rate.

5. Non-SM right-chiral quark current

The quark current with a *right-chiral* piece added can be written as

$$\bar{\psi}(b)\big[\gamma^\mu(1-\gamma_5)\big]\psi(t) \to \bar{\psi}(b)\big[\gamma^\mu(1-\gamma_5)+\alpha\gamma^\mu(1+\gamma_5)\big]\psi(t)$$

where α parametrizes the strength of the right–chiral contribution.

- There is no interference between SM and non-SM currents when $m_b = 0$

The matrix element involving the right-chiral quark current is

$$M = \alpha\ \bar{u}_b\gamma^\mu(1+\gamma_5)u_t\ \bar{u}_\nu\gamma_\mu(1-\gamma_5)v_l$$

For the differential rates one then finally obtains

$$
\begin{aligned}
\frac{d\Gamma_A}{dx} &= \alpha^2 \Gamma_F \, 2\pi \frac{m_W}{\Gamma_W} 6y(x-y)(1-x+y)\,, \\
\frac{d\Gamma_B}{dx} &= \alpha^2 \Gamma_F \, 2\pi \frac{m_W}{\Gamma_W} \frac{6y(x-y)}{x} (2y - x(1+y-x)) \\
\boxed{\frac{d\Gamma_C}{dx}} &= \alpha^2 \Gamma_F \, 2\pi \frac{m_W}{\Gamma_W} \frac{12y(x-y)}{x} \sqrt{y(1-x)(x-y)}\,,
\end{aligned}
$$

The integrated rates are ($y \leq x \leq 1$)

$$
\begin{aligned}
\Gamma_A &= \alpha^2 \Gamma_F \, 2\pi \frac{m_W}{\Gamma_W} y(1 - 3y^2 + 2y^3)\,, \\
\Gamma_B &= \alpha^2 \Gamma_F \, 2\pi \frac{m_W}{\Gamma_W} y(-1 + 12y - 9y^2 - 2y^3 + 12y^2 \ln y)\,, \\
\boxed{\Gamma_C} &= \alpha^2 \Gamma_F \, 2\pi \frac{m_W}{\Gamma_W} \frac{3}{2} \pi y^{3/2} (1 - 6y + 8y^{3/2} - 3y^2)\,.
\end{aligned}
$$

- An interesting exercise: $\alpha \sim 0.0137$ when the magnitude of the *SM NLO* and the *non-SM LO* contributions to Γ_C are the same.

6. Conclusions

For the decay $t(\uparrow) \to b + W^+(\ell^+\nu)$:

- The LO azimuthal correlation function vanishes.
- The NLO contribution to the azimuthal correlation is very small.
- A LO azimuthal correlation is possible with non-SM type of interaction.
- Precision measurements for the azimuthal correlation can be important in understanding the SM.