Vacua and walls of mass-deformed Kähler nonlinear sigma models on $SO(2N)/U(N)$ and $Sp(N)/U(N)$

Masato Arai & SS, PRD83, 125003 (2011)
Bum-Hoon Lee & Chanyong Park & SS, PRD96, 10517 (2017)
Masato Arai & Anastasia Golubtsova & Chanyong Park & SS, PRD97, 10512 (2018)

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Motivation

- Embedding a quadratic constraint into the moduli matrix formalism.
- $\mathcal{N}$=2 supersymmetry formalism: harmonic superspace, projective superspace $\Rightarrow$ supersymmetric vacua of the quadrics of the Grassmann manifold?
- Duality between the root system of B-series and the root system of C-series
  \# of simple roots of $SO(2N+1)$=\# of simple roots of $SO(2N)$
  $\Rightarrow$ Duality between $SO(2N)/U(N)$ and $Sp(N)/U(N)$?

What we know $\cdots$

- The strong gauge coupling limit of $\mathcal{N}=2$ $U(N_C)$ gauge theory in four dimensions ($N_F > N_C$) becomes massive hyper-Kähler nonlinear sigma models on the cotangent bundle over G_{N_F,N_C} [Y.Isozumi& M.Nitta & K.Ohashi & N.Sakai (2004)].
- It is shown that the vector multiplet part of BPS eq. does not produce additional moduli parameters. It is proven in the case of compact Kähler base space and domain walls in $U(1)$ and non-Abelian gauge theories, etc. [Mundet I Riera(2000)], [Cieliebak, Rita Gaio, Salamon(2000)], [Sakai,Yang(2005)], [Sakai,Tong(2005),K.S.M.Lee(2003)].

What we know $\cdots$ (cont'd)

- Moduli matrices of walls on the Grassmann manifold [Y.Isozumi& M.Nitta & K.Ohashi & N.Sakai (2004)].
- Moduli matrices of walls on $SO(2N)/U(N)$ ($N \leq 3$) [M.Arai & SS (2011)].
- Moduli matrices of magnetic monopoles on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ [M.Eto& T.Fujimori & S.B.Gudnason & Y.Jiang & K.Konishi & M.Nitta & K.Ohashi (2011)].
- Moduli matrices of walls on $SO(2N)/U(N)$ [B-H. Lee & C. Park & SS (2017)], moduli matrices of walls on $Sp(N)/U(N)$ [M. Arai & A. Golubtsova & C. Park & SS (2018)]

Lagrangian

•$\mathcal{N}=1$ Lagrangian in 4D [Higashijima & Nitta (1999)]

$$\mathcal{L}=\int d^4\theta\left(\Phi_a{}^i\bar{\Phi}_i{}^b(e^V)_b{}^a-\zeta V_a{}^a\right)+\int d^2\theta\left(\Phi_0^{ab}\left(\Phi_b{}^i J_{ij}\Phi^{Tj}{}_a\right)+(\mathrm{h.c})\right)$$

$$J=\begin{pmatrix}0 & I_N\\ \epsilon I_N & 0\end{pmatrix},\quad \epsilon=\begin{cases}1 & SO(2N)/U(N)\\ -1 & Sp(N)/U(N)\end{cases}$$

$i,j=1,\cdots,2N$, $a,b=1,\cdots,N$
Φ : $N\times 2N$ chiral superfield
V : $N\times N$ vector superfield
Φ_0 : chiral superfield, $\Phi_0^T=\epsilon\Phi_0$

$$\Phi_a{}^i(y,\theta)=\phi_a{}^i(y)+\sqrt{2}\theta\psi_a{}^i(y)+\theta\theta F_a{}^i(y)$$

$$V_a{}^b(x,\theta,\bar{\theta})=2(\theta\sigma^m\bar{\theta})A_{ma}{}^b(x)+i(\theta\theta)(\bar{\theta}\bar{\lambda})_a{}^b(x)-i(\bar{\theta}\bar{\theta})(\theta\lambda)_a{}^b(x)+(\theta\theta)(\bar{\theta}\bar{\theta})D_a{}^b(x)$$

$$\Phi_0^{ab}(y,\theta)=\phi_0^{ab}(y)+\sqrt{2}\theta\psi_0^{ab}(y)+\theta\theta F_0^{ab}(y)$$

Lagrangian (cont'd)

•Lagrangian in the Grassmann manifold

$$G_{N,M} = U(N)/U(N-M) \times U(M), \quad (N > M)$$

$$\mathcal{L} = \int d^4\theta \, \mathrm{Tr}\left(\Phi\Phi^\dagger e^V - \zeta V\right), \; e^V = \frac{\zeta}{\Phi\Phi^\dagger}$$

$$K(\Phi, \Phi^\dagger) = \zeta \, \mathrm{Tr} \ln \Phi\Phi^\dagger = \zeta \ln \det \Phi\Phi^\dagger$$

Gauge fixing

$$\Phi = \begin{pmatrix} I_M & \varphi \end{pmatrix}$$

φ : $M \times (N-M)$ matrix-valued chiral superfield

$$K(\Phi, \Phi^\dagger) = \zeta \ln \det(I_M + \varphi\varphi^\dagger)$$

Kähler potential of the Grassmann manifold
[Zumino (1979)][Alvarez-Gaumé & Freedman (1981)]

Lagrangian

•mass-deformed 3D Lagrangian

$$
\begin{aligned}
&\mathcal{L}_{\rm 3D} = \\
&= -(\overline{D_\mu\phi})_i{}^a (D^\mu\phi)_a{}^i - |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b\phi_b{}^i|^2 + |F_a{}^i|^2 + \frac{1}{2}(D_a{}^b\phi_b{}^i\bar{\phi}_i{}^a - D_a{}^a) \\
&+\Big[(F_0)^{ab}\phi_b{}^i J_{ij}\phi^{Tj}{}_a + (\phi_0)^{ab}F_b{}^i J_{ij}\phi^{Tj}{}_a + (\phi_0)^{ab}\phi_b{}^i J_{ij}F^{Tj}{}_a + c.c.\Big] \\
&(\mu = 0,1,2, \quad i,j = 1,\cdots,2N, \quad a,b = 1,\cdots,N)
\end{aligned}
$$

$$
\begin{array}{rl}
\text{D-term constraint} & \phi_a{}^i\bar{\phi}_i{}^b - \delta_a{}^b = 0 \\
\text{F-term constraints} & \phi_a{}^i J_{ij}\phi^{Tj}{}_b = 0 \quad + \quad \text{complex conj.}
\end{array}
$$

covariant derivative : $(D_\mu\phi)_a{}^i = \partial_\mu\phi_a{}^i - iA_{\mu a}{}^b\phi_b{}^i$

•J: invariant tensor

$$
J = \begin{cases} \sigma^1 \otimes I_N & SO(2N) \\ i\sigma^2 \otimes I_N & USp(2N) \end{cases}
$$

•potential term

$$
V = |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b\phi_b{}^i|^2 + 4|(\phi_0)^{ab}\phi_b{}^i|^2
$$

Lagrangian (cont'd)

•mass matrix
(convention and formalism used in [M.Eto& T.Fujimori & S.B.Gudnason & Y.Jiang & K.Konishi & M.Nitta & K.Ohashi (2011)])

In this basis, Cartan generators are

$$H_K = e_{K,K} - e_{N+K,N+K}, \quad (K = 1, \cdots, N),$$

where $e_{A,A}$ is a $2N \times 2N$ matrix of which the (A, A) component is one.

$$\underline{m} := (m_1, m_2 \cdots, m_N)$$
$$m_1 > m_2 > \cdots > m_N \quad \text{w/o loss of generality}$$

$$\underline{H} := (H_1, H_2, \cdots, H_N)$$

$$M = \underline{m} \cdot \underline{H}$$

Vacua

•vacuum

$$\phi_a^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i = 0$$
$$(\phi_0)^{ab} = 0$$

Σ can be diagonalized by $U(N)$ transformation

$$\Sigma = \mathrm{diag}(\Sigma_1, \Sigma_2, \cdots, \Sigma_N),$$

therefore the vacua are labelled by

$$(\Sigma_1, \Sigma_2, \cdots, \Sigma_N) = (\pm m_1, \pm m_2, \cdots, \pm m_N).$$

$$\rightarrow \# \text{ of vacua} = \begin{cases} 2^{N-1} & (SO(2N)/U(N)) \\ 2^N & (Sp(N)/U(N)) \end{cases}$$

$\#$ of vacua $\to$ Euler's characteristics

[E.Witten (1982)][K.Hori& C.Vafa (2000)][C.U.Sanchez& A.L.Cali & J.L.Moreschi (1997)][S.B.Gudnason & Y.Jiang & K.Konishi (2010)]

orientational moduli space $\mathcal{M}$	$\chi(\mathcal{M})$
$\frac{SO(2N)}{U(N)}$	2^{N-1}
$\frac{Sp(N)}{U(N)}$	2^N
$\mathbf{C}P^{N-1} = \frac{SU(N)}{SU(N-1)\times U(1)}$	N
$Gr_{N,k} = \frac{SU(N)}{SU(N-k)\times SU(k)\times U(1)}$	${}_N C_k$
$Q^{2N-2} = \frac{SO(2N)}{SO(2N-2)\times SO(2)}$	$2N$

BPS equation

The BPS equation for wall solutions is derived from the Bogomol'nyi completion of the Hamiltonian. It is assumed that fields are static and all the fields depend only on the $x_1 \equiv x$ coordinate. It is also assumed that there is Poincare invariance on the two-dimensional world volume of walls to set $A_0 = A_2 = 0$. The energy is saturated when

$$(D\phi)_a{}^i \mp (\phi_a{}^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i) = 0.$$

We choose the upper sign for the BPS equation without loss of generality.

Moduli matrices

•BPS equation

$$(D\phi)_a{}^i - (\phi_a{}^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i) = 0$$

By introducing complex matrix functions $S_a{}^b(x)$ and $f_a{}^i(x)$ defined by

$$\Sigma_a{}^b - iA_a{}^b \equiv (S^{-1}\partial S)_a{}^b,\ \phi_a{}^i \equiv (S^{-1})_a{}^b f_b{}^i,$$

the BPS eq. is solved as

$$\phi_a{}^i = (S^{-1})_a{}^b H_{0b}^j (e^{Mx})_j{}^i.$$

H_0 : moduli matrix

All the quantities are invariant under the transformation

$$S'_a{}^b = V_a{}^c S_c{}^b, \quad H'_{0a}{}^i = V_a{}^c H_{0c}^i, V \in GL(N, \mathbf{C}).$$

The V defines an equivalent class of (S, H_0). $\rightarrow$worldvolume symmetry [Y.Isozumi & M.Nitta & K.Ohashi & N.Sakai(2004)].

Moduli matrices (cont'd)

•constraints

$$\begin{array}{c} \phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0 \\ \phi_a{}^i J_{ij} \phi^{Tj}{}_b = 0 \end{array} \quad \rightarrow \quad \begin{array}{c} H_{0a}^{\ i} (e^{2Mx})_i{}^j H_{0j}^{\dagger}{}^{b} = (S\bar{S})_a{}^b \equiv \Omega_a{}^b \\ H_{0a}^{\ i} J_{ij} H_b^{Tj} = 0 \end{array}$$

•moduli space

$$\begin{array}{l} H'_{0a}{}^{i} = V_a{}^c H_{0c}^{\ i}, V \in GL(N, \mathbf{C}) \\ H_{0a}^{\ i} J_{ij} H_b^{Tj} = 0 \end{array}$$

$\rightarrow$ Moduli space is $SO(2N)/U(N)$ or $Sp(N)/U(N)$.

Elementary walls

[Isozumi& Nitta& Ohashi& Sakai(2004)] [M.Arai,SS(2011)][M.Eto & S.B.Gudnason & Y.Jiang & K.Konishi & M.Nitta & K.Ohashi (2011)][B-H.Lee & C.Park & SS (2017)]

$$[cM, E_i] = c(\underline{m} \cdot \underline{\alpha}_i)E_i = T_{\langle i \leftarrow i+1 \rangle} E_i$$

$\underline{\alpha}_i$: simple roots of $SO(2N)$ or $USp(2N)$
$T_{\langle i \leftarrow i+1 \rangle}$: tension density
$\underline{m} := (m_1, \cdots, m_N)$
$m_i > m_{i+1}$
$\rightarrow$ $\underline{m}$ is a vector in the interior of the positive Weyl chamber,

$$\underline{m} \cdot \underline{\alpha}_i > 0.$$

Cartan generators and simple roots

- $SO(2N)$

$$
\begin{aligned}
E_i &= e_{i,i+1} - e_{i+N+1,i+N}, \quad (i = 1, \cdots, N-1),\\
E_N &= e_{N-1,2N} - e_{N,2N-1},\\
\underline{\alpha}_i &= \hat{e}_i - \hat{e}_{i+1},\\
\underline{\alpha}_N &= \hat{e}_{N-1} + \hat{e}_N.
\end{aligned}
$$

- $USp(2N)$

$$
\begin{aligned}
E_i &= e_{i,i+1} - e_{i+N+1,i+N}, \quad (i = 1, \ldots, N-1),\\
E_N &= e_{N,2N},\\
\underline{\alpha}_i &= \hat{e}_i - \hat{e}_{i+1},\\
\underline{\alpha}_N &= 2\hat{e}_N.
\end{aligned}
$$

- normalization

$$
\begin{aligned}
&\mathrm{Tr}(H_I H_J) = 2\delta_{IJ}, \quad (I, J = 1, \cdots, N),\\
&\mathrm{Tr}(H_I E_i) = 0,\\
&\mathrm{Tr}(E_i E_i^\dagger) = \frac{4}{\underline{\alpha}_i \cdot \underline{\alpha}_i}.
\end{aligned}
$$

Walls

- elementary walls

$$H_{0\langle a\leftarrow b\rangle} = H_{0\langle a\rangle} e^{E_i(r)}, \quad E_i(r) \equiv e^r E_i, \ (i = 1, \cdots, N),$$

- compressed wall with a level n in the nonlinear sigma models on $SO(2N)/U(N)$ and $Sp(N)/U(N)$ with $N_f = 1, \cdots, N-1$

$$H_{0\langle a\leftarrow b\rangle} = H_{0\langle a\rangle} e^{[E_{i_1},[E_{i_2},[E_{i_3},\cdots,[E_{i_n},E_{i_{n+1}}]\cdots]]](r)},$$
$$(i_m = 1, \cdots, N; m = 1, \cdots, n+1)$$

- compressed wall in the nonlinear sigma models on $Sp(N)/U(N)$ with $N_F = N-1, N$

$$H_{0\langle a\leftarrow b\rangle} = H_{0\langle a\rangle} e^{[E_{N-1},[E_{N-1},E_N]](r)} \quad \text{or}$$
$$H_{0\langle a\leftarrow b\rangle} = H_{0\langle a\rangle} e^{[[E_N,E_{N-1}],E_{N-1}](r)}$$

- multiwalls

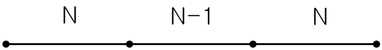
$$H_{0\langle a\leftarrow a'\leftarrow\cdots\leftarrow b\rangle} = H_{0\langle a\rangle} e^{E_{i_1}(r_1)} e^{E_{i_2}(r_2)} \cdots e^{E_{i_n}(r_n)}$$

- Walls are penetrable if

$$[E_{i_m}, E_{i_n}] = 0.$$

Corresponding roots $\Rightarrow \underline{g}_{i_m} \cdot \underline{g}_{i_n} = 0$

$Sp(N)/U(N)$



$$[E_N,[E_{N-1},[E_{N-1},E_N]]]=0,$$
$$\underline{a}_N\cdot(2\underline{a}_{N-1}+\underline{a}_N)=0$$

The three elementary walls cannot be compressed to a compressed wall of level two.
$\Rightarrow$ No duality between $SO(2N)/U(N)$ and $Sp(N)/U(N)$ with non-degenerate mass parameters.

$SO(2N)/U(N)$

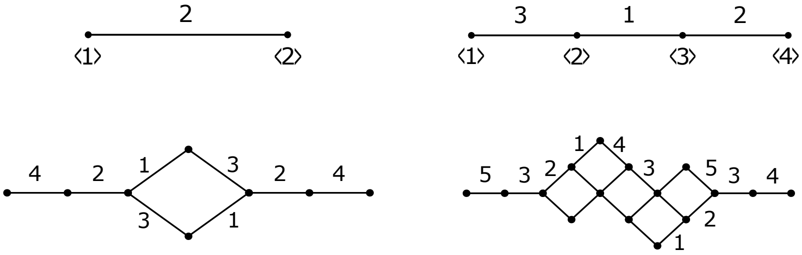


Figure 1: Vacua and elementary walls of nonlinear sigma models on $SO(2N)/U(N)$, $N = 2, 3, 4, 5$. The numbers indicate the subscript i's of roots $\underline{\alpha}_i$. The left-hand side is the limit as $x \to +\infty$ and the right-hand side is the limit as $x \to -\infty$.

$SO(2N)/U(N)$ (cont'd)

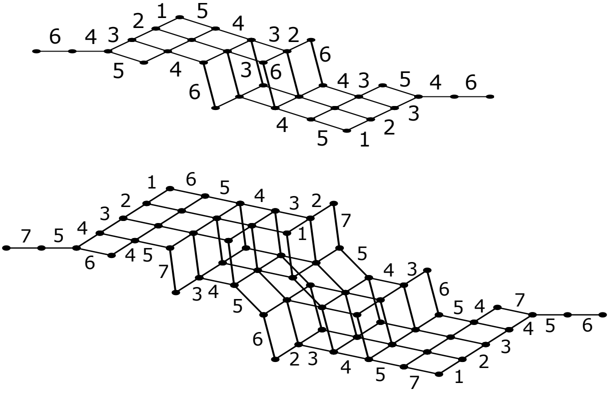


Figure 2: Vacua and elementary walls of nonlinear sigma models on $SO(2N)/U(N)$, $N = 6, 7$. The numbers indicate the subscript i's of roots $\underline{\alpha}_i$.

$SO(N)/U(N)$: vacua connected to the maximum number of elementary walls obtained by induction

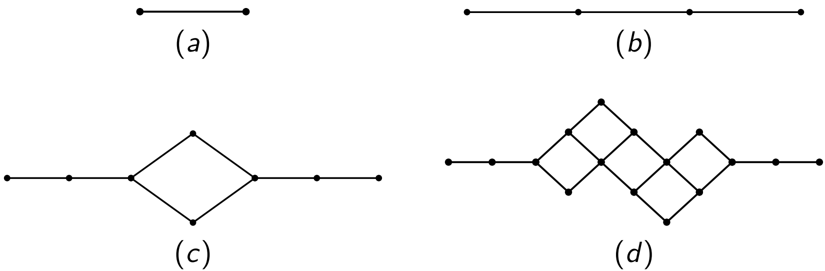


Figure 3: (a)$N = 4m - 2$ (b)$N = 4m - 1$ (c)$N = 4m$ (d)$N = 4m + 1$ $(m \geq 1)$

$SO(N)/U(N)$: vacua connected to the maximum number of elementary walls obtained by induction (cont'd)

$\bullet N = 4m-2$ $(m \geq 2)$

$$\underline{N} \leftarrow \cdots \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-4}}_{(2m-2)} \right\} \leftarrow \langle A \rangle \leftarrow \left\{ \underbrace{\underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-5}}_{(2m-2)}, \underline{4m-2}}_{(2m-1)} \right\} \leftarrow \cdots$$

$$\cdots \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-5}, \underline{4m-2}}_{(2m-1)} \right\} \leftarrow \langle B \rangle \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-4}}_{(2m-2)} \right\} \leftarrow \cdots \leftarrow \underline{N}$$

$\bullet N = 4m-1$ $(m \geq 2)$

$$\underline{N} \leftarrow \cdots \leftarrow \left\{ \underbrace{\underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-4}}_{(2m-2)}, \underline{4m-1}}_{(2m-1)} \right\} \leftarrow \langle A \rangle \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-5}, \underline{4m-3}}_{(2m-1)} \right\} \cdots$$

$$\cdots \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-5}, \underline{4m-3}}_{(2m-1)} \right\} \leftarrow \langle B \rangle \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, 4m-2}_{(2m-1)} \right\} \leftarrow \cdots \leftarrow \underline{N-1}$$

$SO(2N)/U(N)$: vacua connected to the maximum number of elementary walls obtained by induction (cont'd)

$\bullet N = 4m \; (m \geq 2)$

$$\underline{N} \leftarrow \cdots \leftarrow \left\{ \underbrace{\underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-4}}_{(2m-2)}, \underline{4m-1}}_{(2m-1)} \right\} \leftarrow \langle A \rangle \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-5}, \underline{4m-3}}_{(2m-1)} \right\} \cdots$$

$$\cdots \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-5}, \underline{4m-3}}_{(2m-1)} \right\} \leftarrow \langle B \rangle \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-2}}_{(2m-1)} \right\} \leftarrow \cdots \leftarrow \underline{N-1}$$

$\bullet N = 4m + 1 \; (m \geq 2)$

$$\underline{N} \leftarrow \cdots \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m}}_{(2m)} \right\} \leftarrow \langle A \rangle \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-1}}_{(2m)} \right\} \leftarrow \cdots$$

$$\cdots \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-1}}_{(2m)} \right\} \leftarrow \langle B \rangle \leftarrow \left\{ \underbrace{\underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-2}}_{(2m-1)}, \underline{4m+1}}_{(2m)} \right\} \leftarrow \cdots \leftarrow \underline{N-1}$$

$Sp(N)/U(N)$

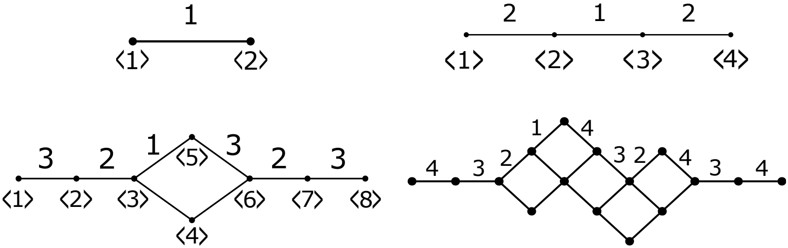


Figure 4: Vacua and elementary walls of nonlinear sigma models on $Sp(N)/U(N)$, $N = 1, 2, 3, 4$. The numbers indicate the subscript i's of roots $\underline{\alpha}_i$. The left-hand side is the limit as $x \to +\infty$ and the right-hand side is the limit as $x \to -\infty$.

$Sp(N)/U(N)$ (cont'd)

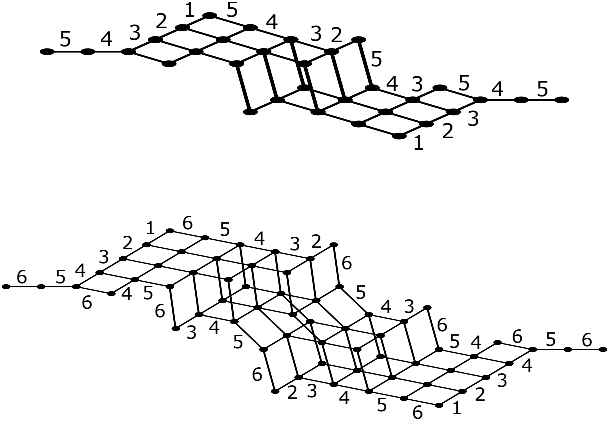


Figure 5: Vacua and elementary walls of nonlinear sigma models on $Sp(N)/U(N)$, $N = 5, 6$.

$Sp(N)/U(N)$: vacua connected to the maximum number of elementary walls obtained by induction

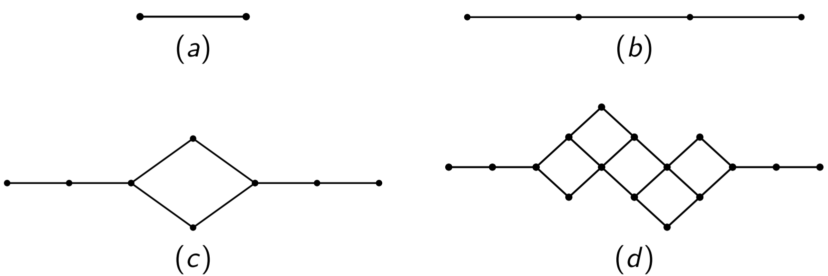


Figure 6: (a)$N = 4m - 3$ (b)$N = 4m - 2$ (c)$N = 4m - 1$ (d)$N = 4m$ $(m \geq 1)$

$Sp(N)/U(N)$: vacua connected to the maximum number of elementary walls obtained by induction (cont'd)

$\bullet N = 4m-3$ $(m \geq 2)$

$$\underline{N} \leftarrow \cdots \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-4}}_{2m-2} \right\} \leftarrow \langle A \rangle \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, , \underline{4m-3}}_{2m-1} \right\} \leftarrow \cdots$$

$$\cdots \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-3}}_{2m-1} \right\} \leftarrow \langle B \rangle \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-4}}_{2m-2} \right\} \leftarrow \cdots \leftarrow \underline{N}$$

$\bullet N = 4m-2$ $(m \geq 2)$

$$\underline{N} \leftarrow \cdots \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-2}}_{2m-1} \right\} \leftarrow \langle A \rangle \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-3}}_{2m-1} \right\} \leftarrow \cdots$$

$$\cdots \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-3}}_{2m-1} \right\} \leftarrow \langle B \rangle \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-2}}_{2m-1} \right\} \leftarrow \cdots \leftarrow \underline{N}$$

$Sp(N)/U(N)$: vacua connected to the maximum number of elementary walls obtained by induction (cont'd)

$\bullet N = 4m-1 \ (m \geq 2)$

$$\underline{N} \leftarrow \cdots \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-2}}_{2m-1} \right\} \leftarrow \langle A \rangle \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-1}}_{2m} \right\} \leftarrow \cdots$$

$$\cdots \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-1}}_{2m} \right\} \leftarrow \langle B \rangle \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m}}_{2m} \right\} \leftarrow \cdots \leftarrow \underline{N}$$

$\bullet N = 4m \ \ (m \geq 2)$

$$\underline{N} \leftarrow \cdots \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, , \underline{4m}}_{2m} \right\} \leftarrow \langle A \rangle \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-1}}_{2m} \right\} \leftarrow \cdots$$

$$\cdots \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-1}}_{2m} \right\} \leftarrow \langle B \rangle \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m}}_{2m} \right\} \leftarrow \cdots \leftarrow \underline{N}$$

Summary

- Moduli matrices for domain walls of nonlinear sigma models on $SO(2N)/U(N)$ and $Sp(N)/U(N)$.

- Pictorial representations where vacua and elementary walls correspond to the vertices and the segments of the representations.
$\Rightarrow$ Full configurations obtained by induction.

- 2^{N-1} vacua in the nonlinear sigma models on $SO(2N)/U(N)$ and 2^N vacua in the nonlinear sigma models on $Sp(N)/U(N)$.

- There are sectors in the nonlinear sigma models on $Sp(N)/U(N)$, which do not allow compressed walls of level two since $\underline{a}_N \cdot (2\underline{a}_{N-1} + \underline{a}_N) = 0$.

Summary

- Moduli matrices for domain walls of nonlinear sigma models on $SO(2N)/U(N)$ and $Sp(N)/U(N)$.

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Thank you !