Sivers function: from small to large transverse momenta

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Outline

- Single Spin Asymmetries in QCD - Sources of Imaginary Phases
- Unsuppressed by 1/Q twist 3
- Non-universality of Sivers function: Colour modification at large -pT
- Sum rules for effective Sivers function from twist 3 effects in spn-dependent DIS
- Sivers function and GPDs
- Conclusions

Single Spin Asymmetries

Main properties:

– Parity: transverse polarization

– Imaginary phase – can be seen T-invariance or technically - from the imaginary i in the (quark) density matrix

Various mechanisms – various sources of phases

Phases in QCD-I

- QCD factorization – soft and hard parts-
- Phases form soft, hard and overlap
- Assume (generalized) optical theorem – phase due to on-shell intermediate states – positive kinematic variable (= their invariant mass)
- Hard: Perturbative (a la QED: Barut, Fronsdal

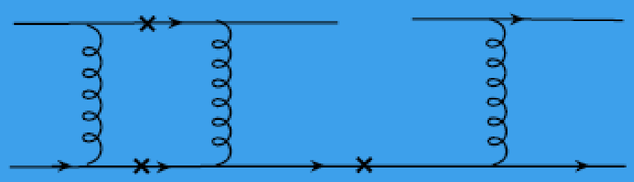
(1960), found at JLAB recently):

Kane, Pumplin, Repko (78) Efremov (78)

Perturbative PHASES IN QCD

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like $q - e$ scattering in DIS):

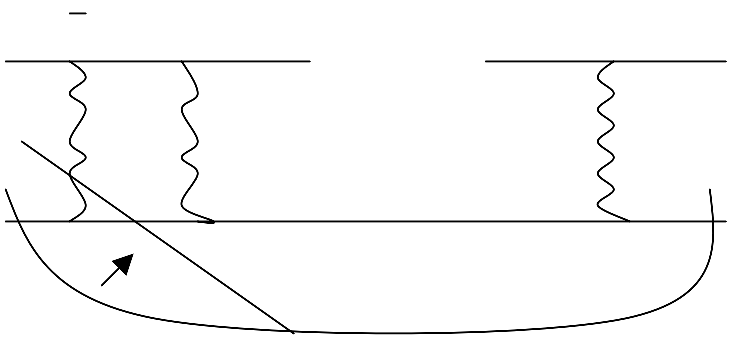


$$A \sim \frac{\alpha_S m p_T}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

Short+ large overlap– twist 3

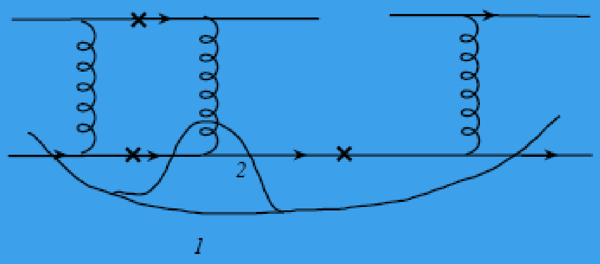
- Quarks – only from hadrons
- Various options for factorization – shift of SH separation



- New option for SSA: Instead of 1-loop twist 2 – Born twist 3: Efremov, OT (85, Ferminonc poles); Qiu, Sterman (91, GLUONIC poles)

Twist 3 correlators

Escape: QCD factorization - possibility to shift the borderline between large and short distances



At short distances - Loop $\rightarrow$ Born diagram

At Large distances - quark distribution $\rightarrow$ quark-gluon correlator.

Physically - process proceeds in the external gluon field of the hadron.

Leads to the shift of α_S to non-perturbative domain AND

"Renormalization" of quark mass in the external field up to an order of hadron's one

$$\frac{\alpha_S m p_T}{p_T^2 + m^2} \rightarrow \frac{M b(x_1, x_2) p_T}{p_T^2 + M^2}$$

Further shift of phases completely to large distances - T-odd fragmentation functions. Leading twist transversity distribution - no hadron mass suppression.

Phases in QCD –large distances in fragmentation

- Non-perturbative - positive variable
- Jet mass-Fragmentation function: Collins(92);Efremov,Mankiewicz, Tornqvist (92),
- Correlated fragmentation: Fracture function: Collins (95), O.T. (98).

Phases in QCD-Large distances in distributions

- Distributions :Sivers, Boer and Mulders – no positive kinematic variable producing phase
- QCD: Emerge only due to (initial of final state) interaction between hard and soft parts of the process: “Effective” or “non-universal” SH interactions by physical gluons – Twist-3 (Boer, Mulders, OT, 97)
- Brodsky -Hwang-Schmidt model:the same SH interactions as twist 3 but non-suppressed by Q: Sivers function – leading (twist 2).

What is “Leading” twist?

- Practical Definition - Not suppressed as M/Q
- However – More general definition: Twist 3 may be suppresses

as M/P_T

. Twist 3 may contribute at leading order

in 1/Q !

Does this happen indeed?? – Explicit calculation

for the case when $Q >> P_T$

May be interesting for experimental studies

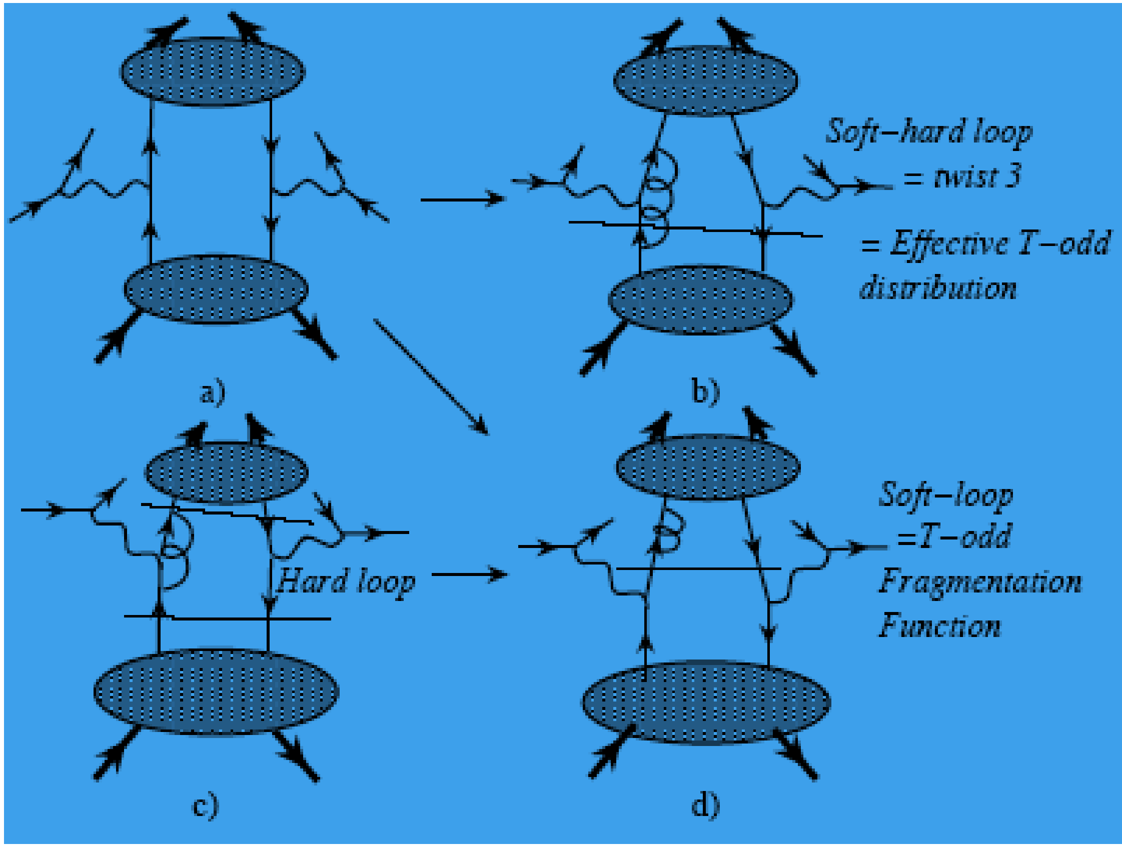
Sources of Phases in SIDIS and Drell-Yan processes

- a) Born - no SSA
- b) -Sivers (can

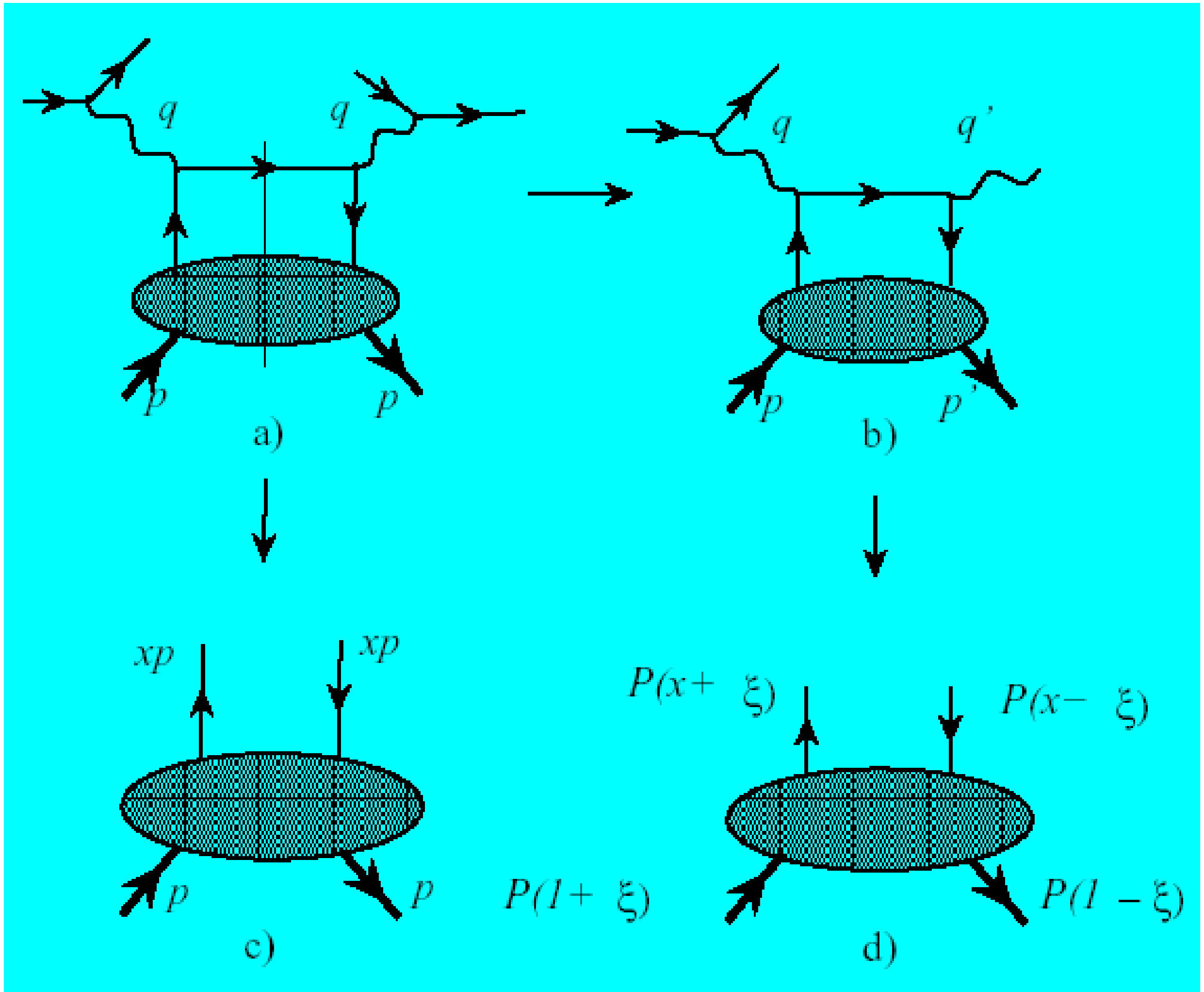
be only effective)

-for both SIDIS and DY

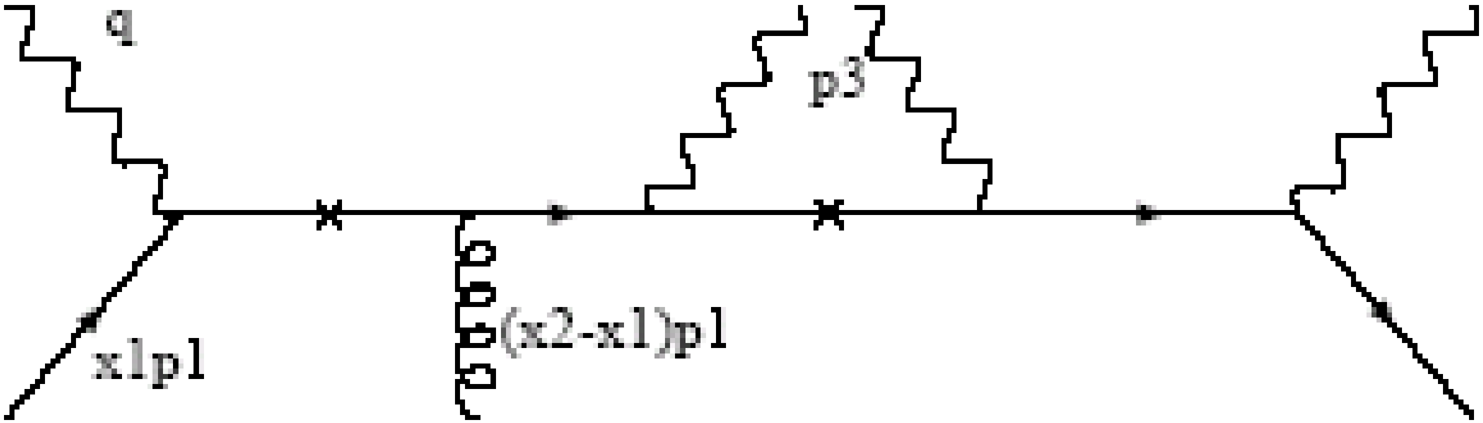
- c) Perturbative
- d) Collins (SIDIS) or (effective) Boer (DY)



Final Pion -> Photon: SIDIS -> SIDVCS (clean, easier than exclusive) - analog of DVCS



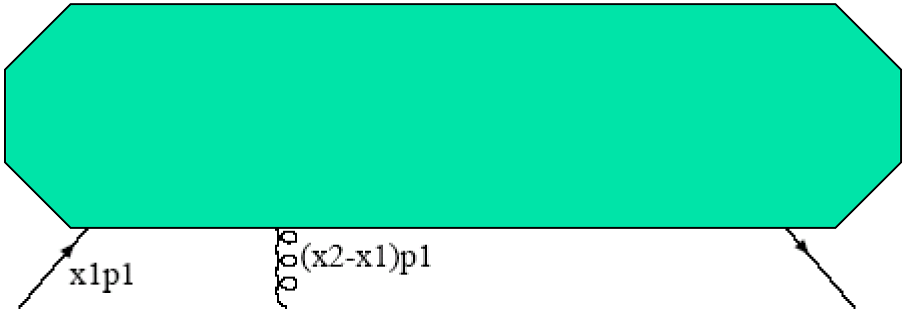
Twist 3 partonic subprocesses for SIDVCS



Real and virtual photons - most clean tests of QCD

- Both initial and final – real :Efremov, O.T. (85)
- Initial – quark/gluon, final - real : Efremov, OT (86, fermionic poles); Qui, Sterman (91, GLUONIC poles)
- Initial - real, final-virtual (or quark/gluon) – Korotkiian, O.T. (94)
- Initial –virtual, final-real: O.T., Srednyak (05; smooth transition from fermionic via hard to GLUONIC poles).

Quark-gluon correlators



- Non-perturbative NUCLEON structure – physically mean the quark scattering in external gluon field of the HADRON.
- Depend on TWO parton momentum fractions
- For small transverse momenta – quark momentum fractions are close to each other- gluonic pole; probed if :
  $Q >> P_T >> M$

$$x_2 - x_1 = \delta = \frac{p_T^2 x_B}{Q^2 z}$$

Cross-sections at low transverse momenta:

$$d\sigma_{total} = f(x_{Bj})8Q^2\frac{x_{Bj}^2(1+(1-y)^2)(1+(1-z)^2)}{y^2 z\delta} \qquad (12)$$

$$d\sigma_{ax1x2} = b_A(x_{Bj},x_2)8Mp_T\frac{x_{Bj}(1+(1-y)^2)(2-z)}{y^2(1-z)\delta}s_T sin(\phi_s^h) \qquad (13)$$

$$d\sigma_{vx1x2} = b_V(x_{Bj},x_2)8Mp_T\frac{x_{Bj}(1+(1-y)^2)(1+(1-z)^2)}{y^2 z(1-z)\delta}s_T sin(\phi_s^h) \qquad (14)$$

$$d\sigma_{a0x2} = -b_A(0,x_2)8Mp_T\frac{x_{Bj}^2(2(1-y)(1-2z)+y^2(1-z))}{y^2z^2\delta}s_T sin(\phi_s^h)$$

(14) - non-suppressed for large Q if Gluonic pole exists=effective Sivers function; spin-dependent looks like unpolarized (soft gluon)

$$A \propto \frac{2M\, p_T \varphi_V(x_B)}{m_T^2 x_B q(x_B)} s_T \sin\phi_h^s$$

Effective Sivers function

- Needs (soft) talk of large and short distances
- Complementary to gluonic exponential, when longitudinal (unsuppressed by Q, unphysical) gluons get the physical part due to transverse link (Belitsky, Ji, Yuan)
- We started instead with physical (suppressed as 1/Q) gluons, and eliminated the suppression for gluonic pole.
- Another support – Ji, Qiu, Vogelsang, Yuan in DY and SIDIS – PERTURBATIVE Sivers function from twist 3

Other way - NP Sivers and gluonic poles at large PT (P.G. Ratcliffe, OT, **hep-ph/0703293**)

- Sivers factorized (general!) expression

$$d\Delta\sigma = \int d^2k_T dx\, f_S(x,k_T)\ \mathrm{Tr}[\gamma^\rho H(x,k_T)] \frac{\epsilon^{\rho s P k_T}}{M}$$

- Expand in kT = twist 3 part of Sivers

$$d\Delta\sigma = \int dx\, f_S(x,k_T)\ \mathrm{Tr}\left[\gamma^\rho \frac{\partial H(x,k_T=0)}{\partial k_T^\alpha} k_T^\alpha\right] \epsilon^{\rho s P k_T}$$

From Sivers to twist 3 - II

- Angular average : $$\langle k_T^\mu k_T^\nu \rangle = -\frac{g_T^{\mu\nu}}{2} \langle k_T^2 \rangle$$

$$g_T^{\mu\nu} = g^{\mu\nu} - P^\mu n^\nu - n^\mu P^\nu$$

- As a result $$d\Delta\sigma = -M \int dx\, f_S^{(1)}(x)\, \mathrm{Tr}\left[\gamma^\rho \frac{\partial H(x, k_T = 0)}{\partial k_T^\alpha}\right]$$

$$f_S^{(1)}(x) = \int d^2 k_T\, f_S(x, k_T)\, \frac{k_T^2}{2M^2}.$$ $$\left(\epsilon^{\rho s P \alpha} - P^\alpha \epsilon^{\rho s P n}\right)$$

- M in numerator - sign of twist 3. Higher moments – higher twists. KT dependent function – resummation of higher twists

From Sivers to gluonic poles - III

- Final step – kinematical identity

$$\epsilon^{\rho s P \alpha} = P^{\alpha}\epsilon^{\rho s P n} - P^{\rho}\epsilon^{\alpha s P n}$$

- Two terms are combined to one

$$d\Delta\sigma = M \int dx\, f_S^{(1)}(x)\ \mathrm{Tr}\left[\gamma \cdot P \frac{\partial H(x, k_T = 0)}{\partial k_T^{\alpha}}\right] \epsilon^{\alpha s P n}$$

- Key observation – exactly the form of Master Formula for gluonic poles (Koike et al, 2007)
- Non – Suppression as 1/Q seen!

Effective Sivers function

- Follows the expression similar to BMT, BMP, JVY

$$x f_S^{(1)}(x) = \sum C_i \frac{1}{2M} T_j(x,x),$$

- Up to Colour Factors !
- Defined by colour charge (natural for low energy theorems!): Collins sign rule: ISI -> -FSI holds because of quark -> antiquark (cf. Abelian charge – Collins&Qiu (**arXiv:0705.2141**)

What are these factors?

- SIDIS at large pT : -1/6 for mesons from quark, 3/2 from gluon fragmentation (kaons?)
- DY at large pT (PAX): 1/6 in quark antiquark annihilation, - 3/2 in gluon Compton subprocess – Collins sign rule more elaborate – involve crossing of distributions and fragmentations - special role of PION DY (COMPASS).
- Hadronic pion production – more complicated – studied for P-exponentials by Amsterdam group + VW
- FSI for pions from quark fragmentation

-1/6 x (non-Abelian Compton) +1/8 x (Abelian Compton)

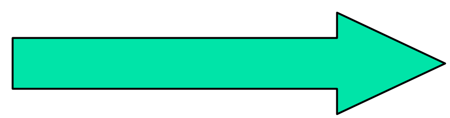
How to pass from high to low PT

- Hard poles in correlators (become soft ay small PT – c.f. SIDVCS)
- Low pT – cannot distinguish fragmentation from quarks and gluons: 3/2-1/6 = 4/3 (Abelian)
- Strong transverse momentum dependence, very different for mesons from quark and gluon fragmentation

Colour flow

- Quark at large PT:-1/6

- Gluon at large PT : 3/2



- Low PT – combination of quark and gluon: 4/3 (absorbed to definition of Sivers function)

- Similarity to colour transparency phenomenon

Twist 3 factorization (Efremov, OT '84, Ratcliffe, Qiu, Sterman)

- Convolution of soft (S) and hard (T) parts

$$d\sigma_s = \int dx_1 dx_2 \frac{1}{4} Sp[S_\mu(x_1,x_2)T_\mu(x_1,x_2)]$$

- Vector and axial correlators: define hard process for both double (g_2) and single asymmetries

$$T_\mu(x_1,x_2) = \frac{M}{2\pi}(\hat{p}_1\gamma^5 s_\mu b_A(x_1,x_2) - i\gamma_\rho \epsilon^{\rho\mu s p_1} b_V(x_1,x_2))$$

Twist 3 factorization -II

- Non-local operators for quark-gluon correlators

$$b_A(x_1,x_2) = \frac{1}{M}\int \frac{d\lambda_1 d\lambda_2}{2\pi} e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \langle p_1,s|\bar{\psi}(0)\hat{n}\gamma^5(D(\lambda_1)s)\psi(\lambda_2)|p_1,s\rangle,$$

$$b_V(x_1,x_2) = \frac{i}{M}\int \frac{d\lambda_1 d\lambda_2}{2\pi} e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \epsilon^{\mu s p_1 n} \langle p_1,s|\bar{\psi}(0)\hat{n}D_\mu(\lambda_1)\psi(\lambda_2)|p_1,s\rangle$$

- Symmetry properties (from T-invariance)

$$b_A(x_1,x_2) = b_A(x_2,x_1),\ \ b_V(x_1,x_2) = -b_V(x_2,x_1)$$

Twist-3 factorization -III

- Singularities

$$b_A(x_1,x_2) = \varphi_A(x_1)\delta(x_1-x_2) + b^r_A(x_2,x_1).$$

$$b_V(x_1,x_2) = \frac{\varphi_V(x_1)}{x_1-x_2} + b^r_V(x_1,x_2)$$

- Very different: for axial – Wandzura-Wilczek term due to intrinsic transverse momentum
- For vector-GLUONIC POLE (Qiu, Sterman '91) – large distance background

Sum rules

- EOM + n-independence (GI+rotational invariance) –relation to (genuine twist 3) DIS structure functions

$$\int_0^1 x^n \bar{g}_2(x) dx = \int_0^1 x^n \left(\frac{n}{n+1} g_1(x) + g_2(x)\right) dx =$$
$$-\frac{1}{\pi(n+1)} \int_{|x_1, x_2, x_1 - x_2| \le 1} dx_1 dx_2 \sum_f e_f^2 [\frac{n}{2} b_V(x_1, x_2)(x_1^{n-1} - x_2^{n-1}) +$$
$$b_A^r(x_1, x_2)\phi_n(x_1, x_2)], \quad \phi_n(x, y) = \frac{x^n - y^n}{x - y} - \frac{n}{2}(x^{n-1} - y^{n-1}), \quad n = 0, 2...$$

Sum rules -II

- To simplify – low moments

$$\int_0^1 x^2 \hat{g}_2(x)dx =$$
$$-\frac{1}{3\pi}\int_{|x_1,x_2,x_1-x_2|\leq 1} dx_1 dx_2 \sum_f e_f^2 b_V(x_1,x_2)(x_1-x_2)$$

- Especially simple – if only gluonic pole kept:

$$\int_0^1 x^2 \bar{g}_2(x)dx = -\frac{1}{3\pi}\int_{|x_1,x_2,x_1-x_2|\leq 1} dx_1 dx_2 \sum_f e_f^2 \varphi_V(x_1)$$
$$= -\frac{1}{3\pi}\int_{-1}^1 dx_1 \sum_f e_f^2 \varphi_V(x_1)(2-|x_1|)$$

Gluonic poles and Sivers function

- Gluonic poles – effective Sivers functions-Hard and Soft parts talk, but SOFTLY

- Implies the sum rule for effective Sivers function (soft=gluonic pole dominance assumed in the whole allowed x's region of quark-gluon correlator)

$$x\, f_T(x) = \frac{1}{2M} T(x,x) = \frac{1}{4} \phi_v(x)$$

$$\int_0^1 dx x^2 \bar{g}_2(x) = \frac{4}{3\pi} \int_0^1 dx x\, f_T(x)(2-x)$$

Compatibility of SSA and DIS

- Extractions of Sivers function: – “mirror” u and d
- First moment of EGMMS = 0.0072 (0.0042 – 0.014)
- Twist -3 g_2 - similar for neutron and proton (0.005) and of the same sign – nothing like mirror picture seen –but supported by colour ordering!
- Current status: Scale of Sivers function – seems to be reasonable, but flavor dependence differs qualitatively.
- Inclusion of pp data, global analysis including gluonic (=Sivers) and fermionic poles

Relation of Sivers function to GPDs

- Qualitatively similar to Anomalous Magnetic Moment (Brodsky et al)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (**hep-ph/0612205**): $x f_T(x) \sim xE(x)$

- Burkardt SR for Sivers functions is now related to Ji SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x f_T(x) = \sum_{q,G} \int dx x E(x) = 0$$

How gravity is coupled to nucleons?

- Current or constituent quark masses ?– neither!
- Energy momentum tensor - like electromagnertic current describes the coupling to photons

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’) – not yet checked
- Anomalous gravitomagnetic moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

Gravitational formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle \quad = \quad \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_q(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity – similar t-dependence to EM FF

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \qquad M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2 e_q P^\mu \qquad \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2 P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2 e_q M \phi(q) \qquad M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle

Gravitomagnetism

- Gravitomagnetic field – action on spin – ½ from $M = \frac{1}{2}\sum_{q,G} \langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$\vec{H}_J = \frac{1}{2} rot \vec{g};\ \vec{g}_i \equiv g_{0i}$ spin dragging twice smaller than EM

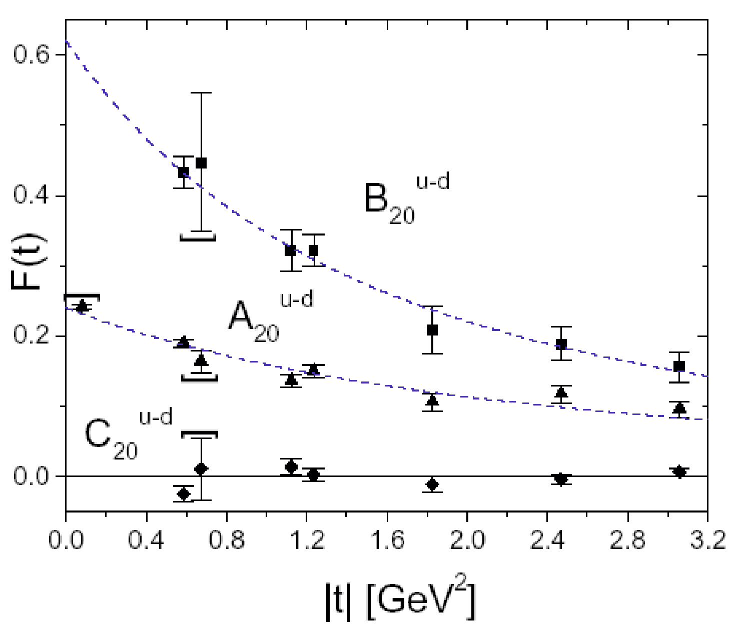
- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$
  Larmor frequency same as EM $\vec{H}_L = rot \vec{g}$
- Orbital and Spin momenta dragging – the same - Equivalence principle $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L$

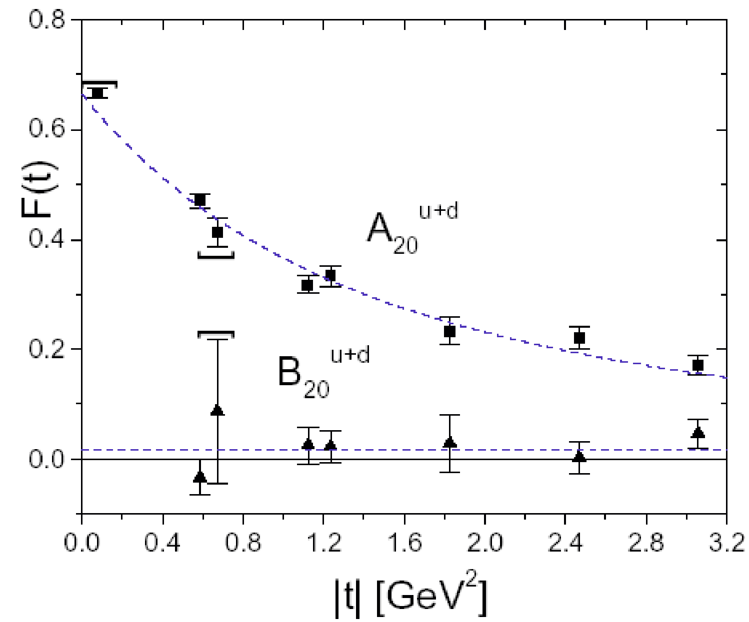
Sivers function and Extended Equivalence principle

- Second moment of E – zero SEPARATELY for quarks and gluons –only in QCD beyond PT (OT, 2001) - supported by lattice simulations etc.. ->
- Gluon Sivers function is small! (COMPASS, STAR, Brodsky&Gardner)
- BUT: gluon orbital momentum is NOT small: total – about 1/2, if small spin – large (longitudinal) orbital momentum
- Gluon Sivers function should result from twist 3 correlator of 3 gluons: remains to be proved!

Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM, confirmed recently)





CONCLUSIONS

- Sivers and other TMD functions contain infinite tower of twists starting from 3 – special role of moments
- Colour charge of initial/final partons crucial – NO factorization in naive sense (cf Abelian model of Collins&Qiu)
- Transverse momentum dependence of Sivers SSA in SIDIS and DY (PAX) is a new sensitive test of QCD
- Relation of Sivers function to twist 3 in DIS: Reasonable magnitude, but problems with flavor dependence.Bochum results with suppressed singlet twist 3 supported!
- Relation of Sivers to GPD’s – link to Nucleon Spin and Equivalence Principle
- Problems: evolution (no WW for Sivers) and SR from twist 3.