Spin-07 3-7 Sept 2007, Dubna

# Spin Structure Function $g_1$ at arbitrary x and $Q^2$

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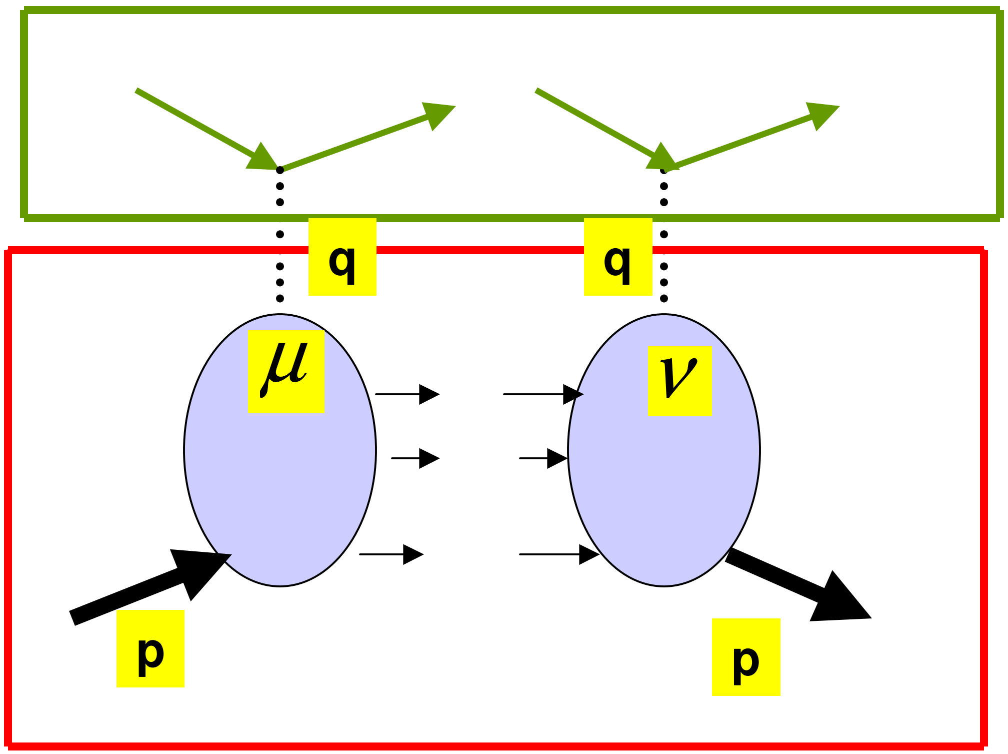
talk based on results obtained in collaboration

with M. Greco and S.I. Troyan




**Leptonic tensor**

**hadronic tensor** $W_{\mu\nu}$

**Does not depend on spin**

**Spin-dependent**

$$W_{\mu\nu} = W_{\mu\nu}^{unpolarized}(p,q) + W_{\mu\nu}^{spin}(p,q)$$

**symmetric**

**antisymmetric**

Spin-dependent part of $W_{\mu\nu}$ is parameterized by two structure functions:

$$W_{\mu\nu}^{spin} = \frac{m}{pq} i\varepsilon_{\mu\nu\lambda\rho} q_\lambda \left[ S_\rho g_1(x,Q^2) + \left( S_\rho - \frac{Sq}{pq} p_\rho \right) g_2(x,Q^2) \right]$$

where m, p and S are the hadron mass, momentum and spin;
q is the virtual photon momentum ($Q^2 = -q^2 > 0$). Again both functions depend on $Q^2$ and $x = Q^2/2pq$, $0 < x < 1$. They measure asymmetries

$g_1$ measures the longitudinal spin flip

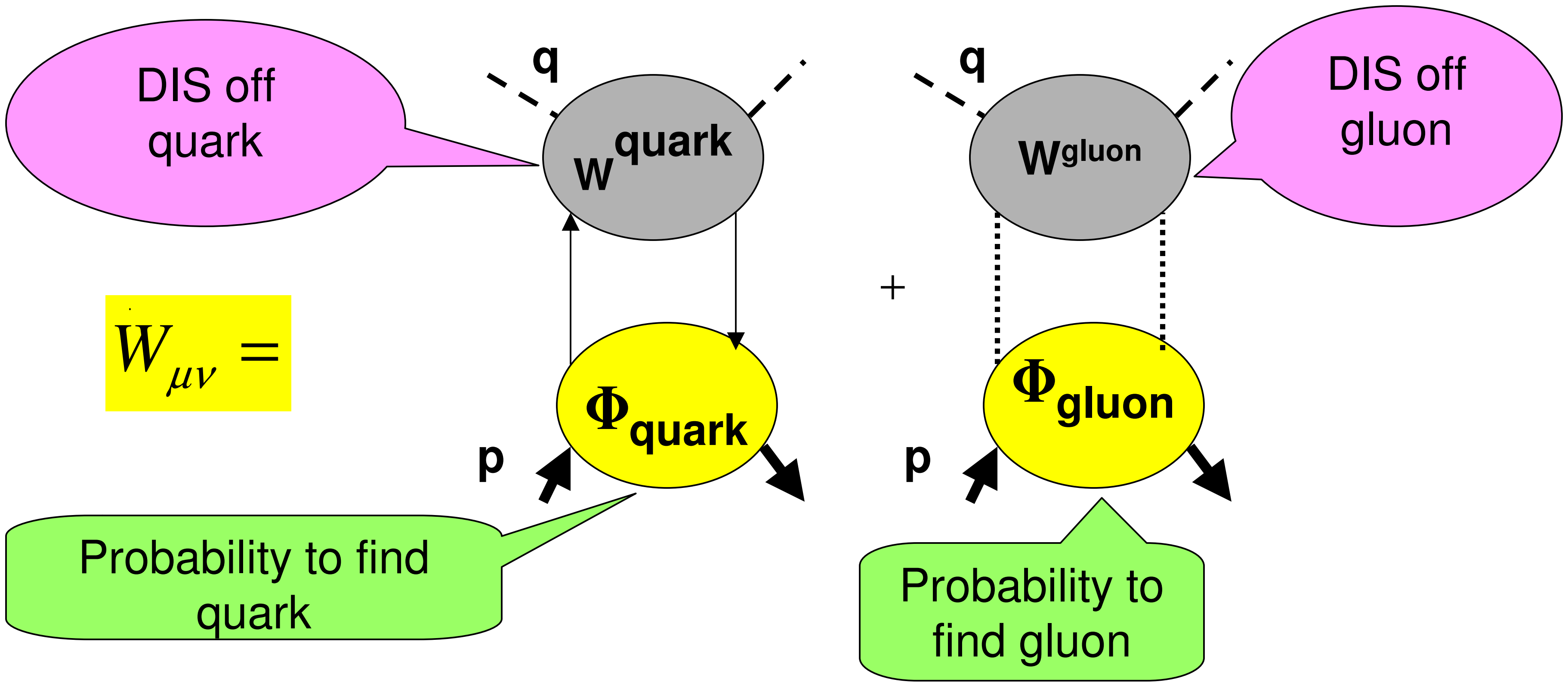
$$g_1 \propto \sigma_{L\uparrow\uparrow} - \sigma_{L\uparrow\downarrow}$$

$g_1 + g_2$ measures the transverse spin flip

$$g_1 + g_2 \propto \sigma_{T\uparrow\uparrow} - \sigma_{T\uparrow\downarrow}$$

**FACTORISATON:** $W_{\mu\nu}$ **is a convolution of the the partonic tensor and probabilities to find a polarized parton (quark or gluon) in the hadron :**

$$W_{\mu\nu} =$$



DIS off quark and gluon can be studied with perturbative QCD, with calculating involved Feynman graphs.

Probabilities, $\Phi_{quark}$ and $\Phi_{gluon}$ involve non-perturbaive QCD. There is no a regular analytic way to calculate them. Usually they are defined from experimental data at large **x** and small **$Q^2$** , they are called the initial quark and gluon densities and are denoted $\delta$**q** and $\delta$**g** .

So, the conventional form of the hadronic tensor is:

$$W_{\mu\nu} = W_{\mu\nu}^{quark} \otimes \delta q + W_{\mu\nu}^{gluon} \otimes \delta g$$

DIS off the quark,

Initial quark distribution

DIS off the gluon

Initial gluon distribution

are calculated with methods of Pert QCD

## Standard Approach

**includes the DGLAP Evolution Equations and the Standard Fits for initial parton densities**

### DGLAP Evolution Equations

**Altarelli-Parisi,Gribov-Lipatov, Dokshitzer**

$$g_1(x,Q^2) = C_q(x/y) \otimes \Delta q(y,Q^2) + C_g(x/y) \otimes \Delta g(y,Q^2)$$

**Evolved quark distribution**

**Evolved gluon distribution**

**Coefficient function**

**Coefficient function**

## DGLAP evolution equations

$$\frac{d\Delta q}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{qq} \otimes \Delta q + \frac{\alpha_s}{2\pi} P_{qg} \otimes \Delta g$$

$$\frac{d\Delta g}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{gq} \otimes \Delta q + \frac{\alpha_s}{2\pi} P_{gg} \otimes \Delta g$$

$P_{qq},\ P_{qg},\ P_{gq}, P_{gg}$ are splitting functions

**Mellin transformation of the splitting functions**
**= anomalous dimensions**

The Standard Approach includes the DGLAP Evolution Equations and the Standard Fits for initial parton densities. One can say that SA combines Science and Art

## SCIENCE

= Calculating splitting functions, anomalous dimensions, coefficient functions

## ART

= the art of composing the fits for initial parton densities

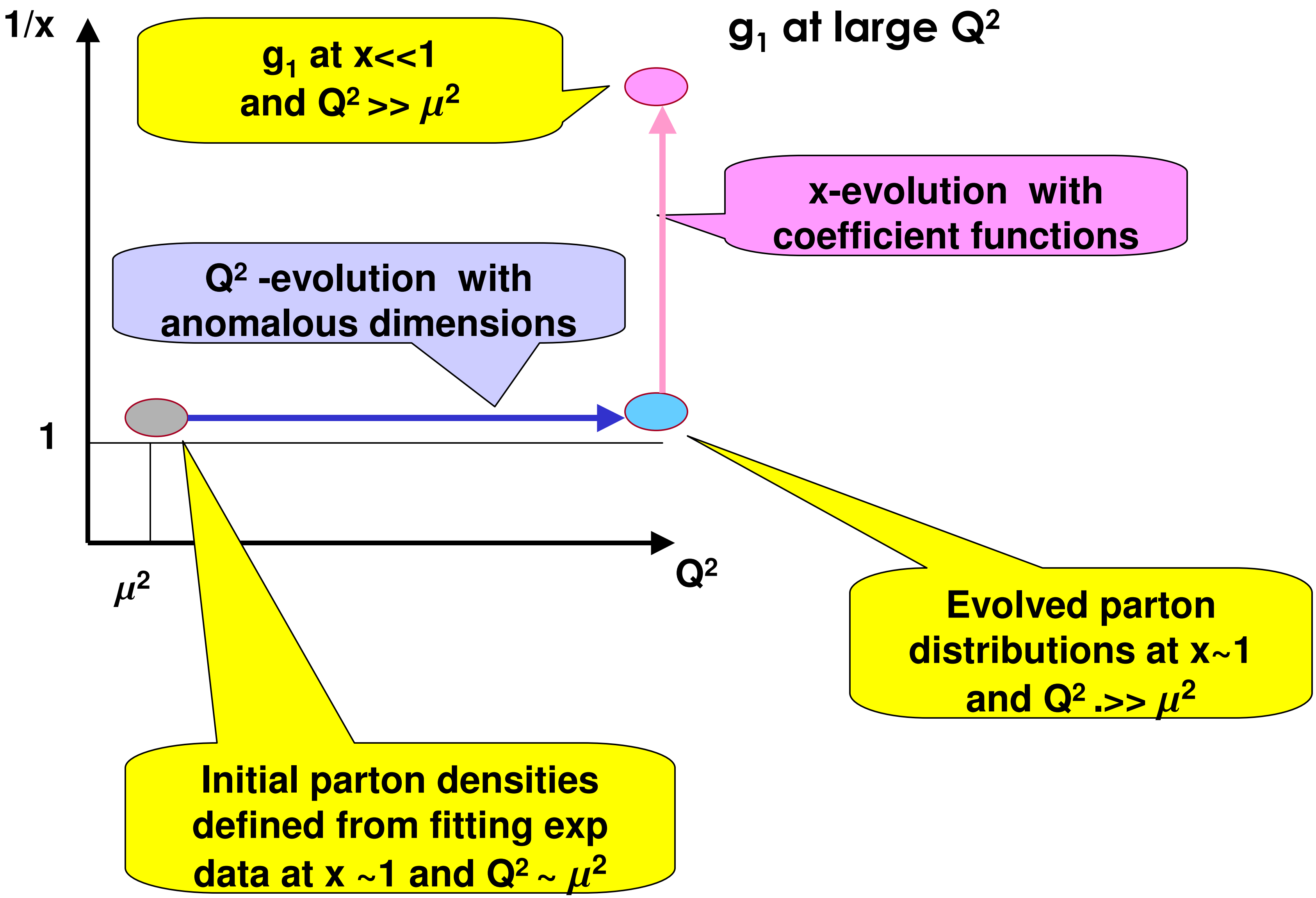
$$\delta q = N x^{-\alpha} [(1 - x)^{\beta} (1 + \gamma x^{\delta})]$$

$$\delta q = N [\ln^{\alpha}(1/x) + \gamma x \ln^{\beta}(1/x)]$$

Altarelli-Ball-Forte-Ridolfi,

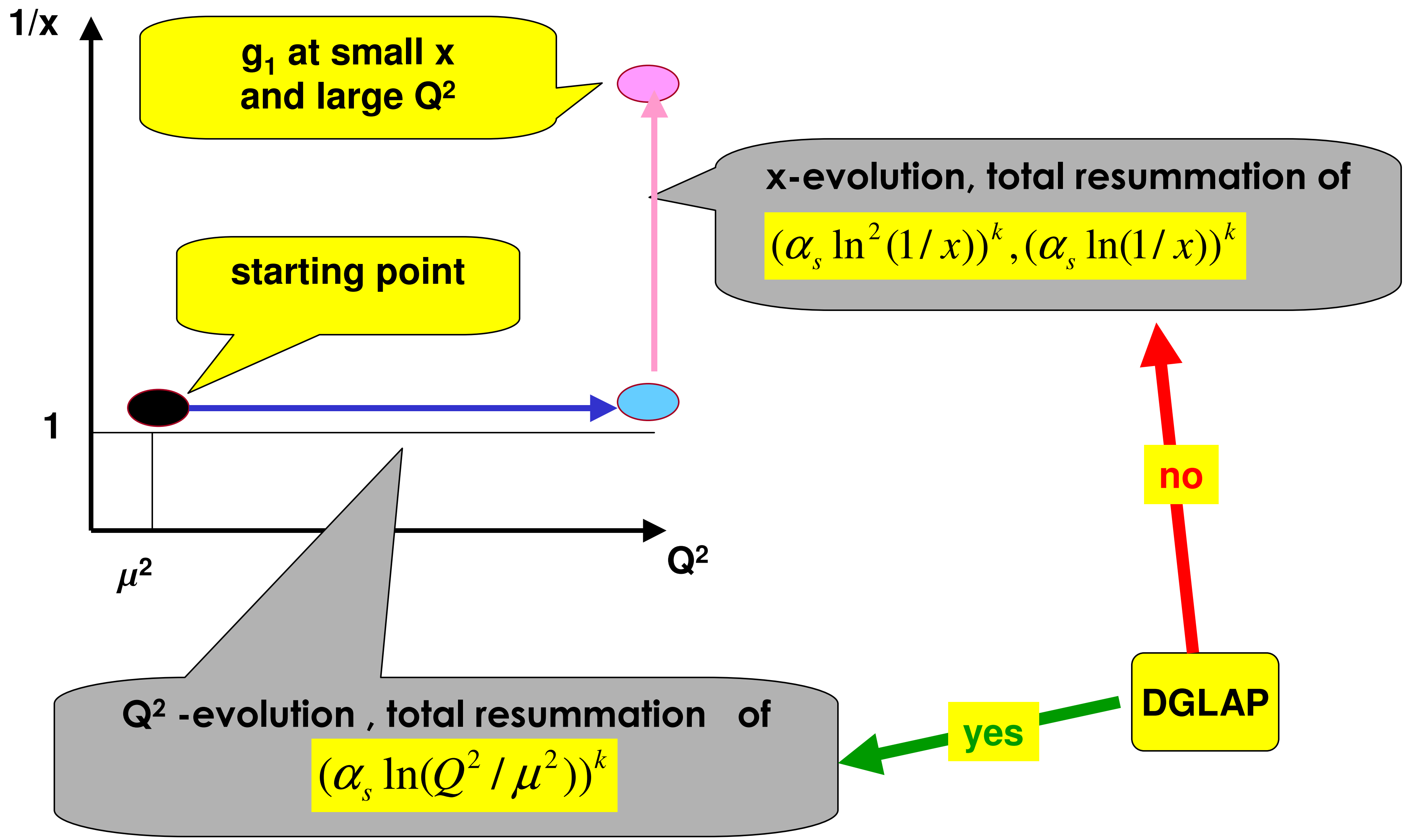
Parameters $N, \alpha, \beta, \gamma, \delta$ should be fixed from experiment

This combination of Science and Art works well at large $Q^2$ and large and even small x. Although from theoretical considerations, DGLAP is not supposed to be used in the small- x region:

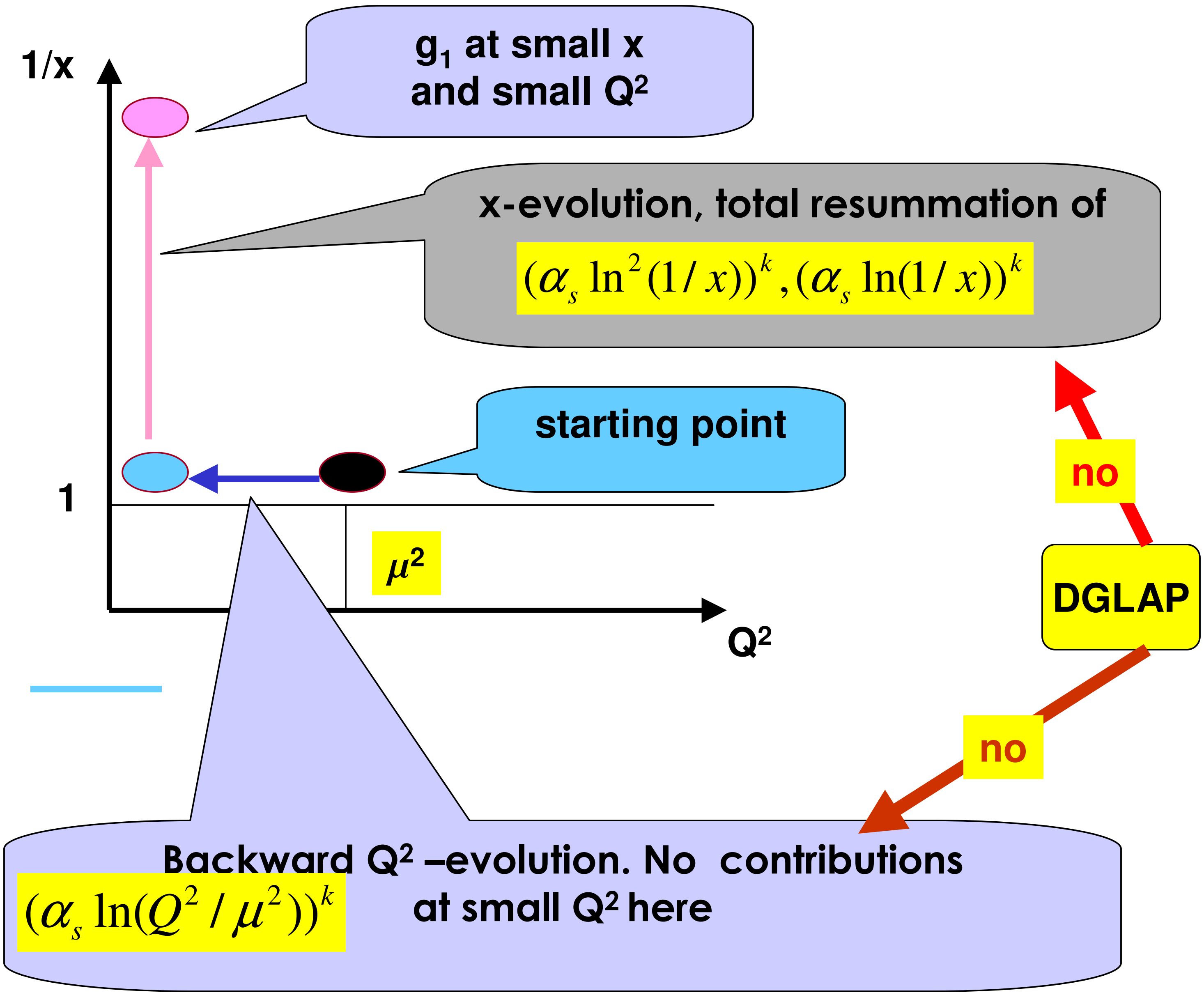


1/x
$g_1$ at large $Q^2$
$g_1$ at x<<1
and $Q^2 >> \mu^2$
x-evolution with
coefficient functions
$Q^2$ -evolution with
anomalous dimensions
1
$\mu^2$
$Q^2$
Evolved parton
distributions at x~1
and $Q^2$ .>> $\mu^2$
Initial parton densities
defined from fitting exp
data at x ~1 and $Q^2 \sim \mu^2$

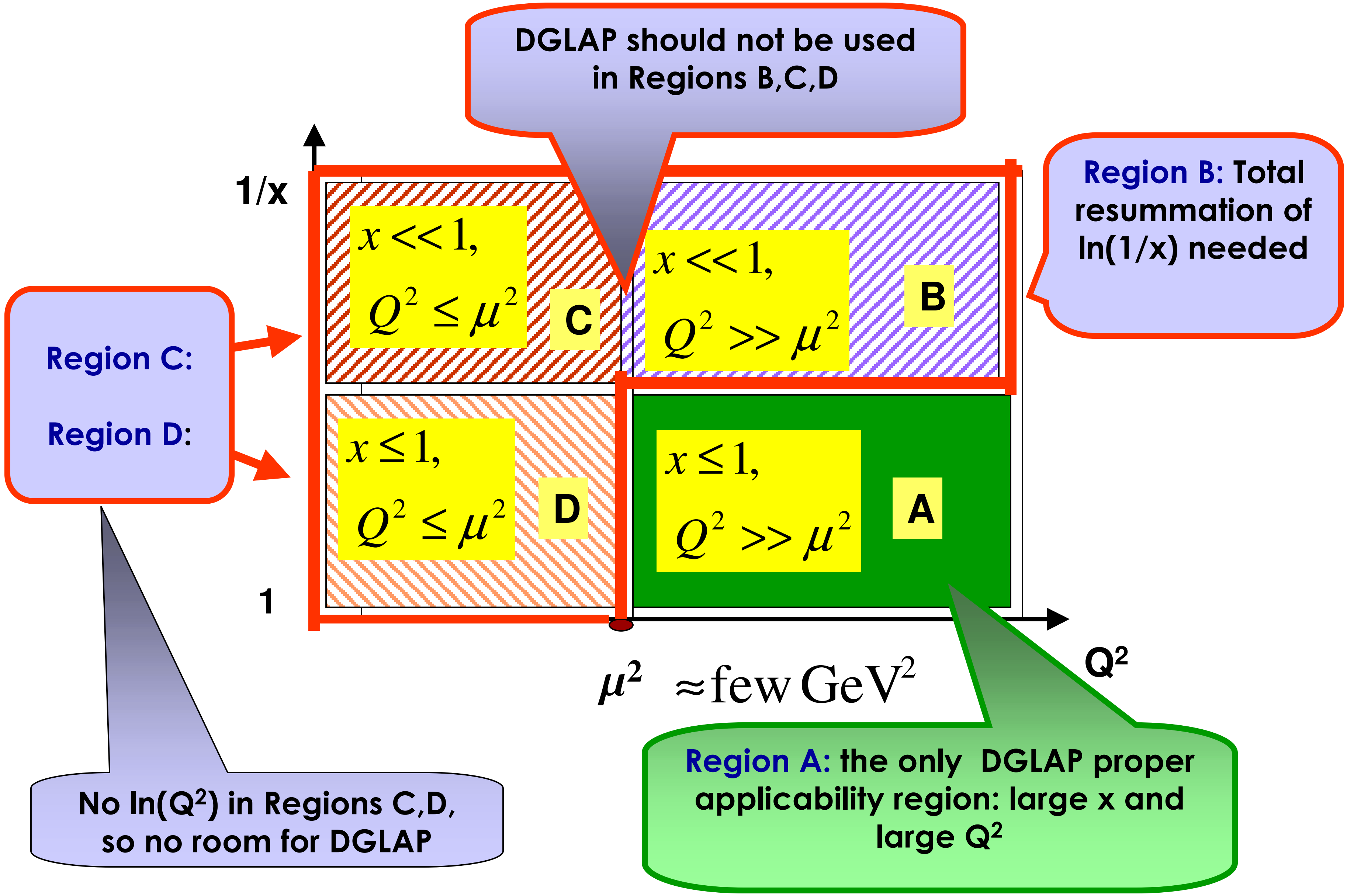
## $g_1$ at large $Q^2$



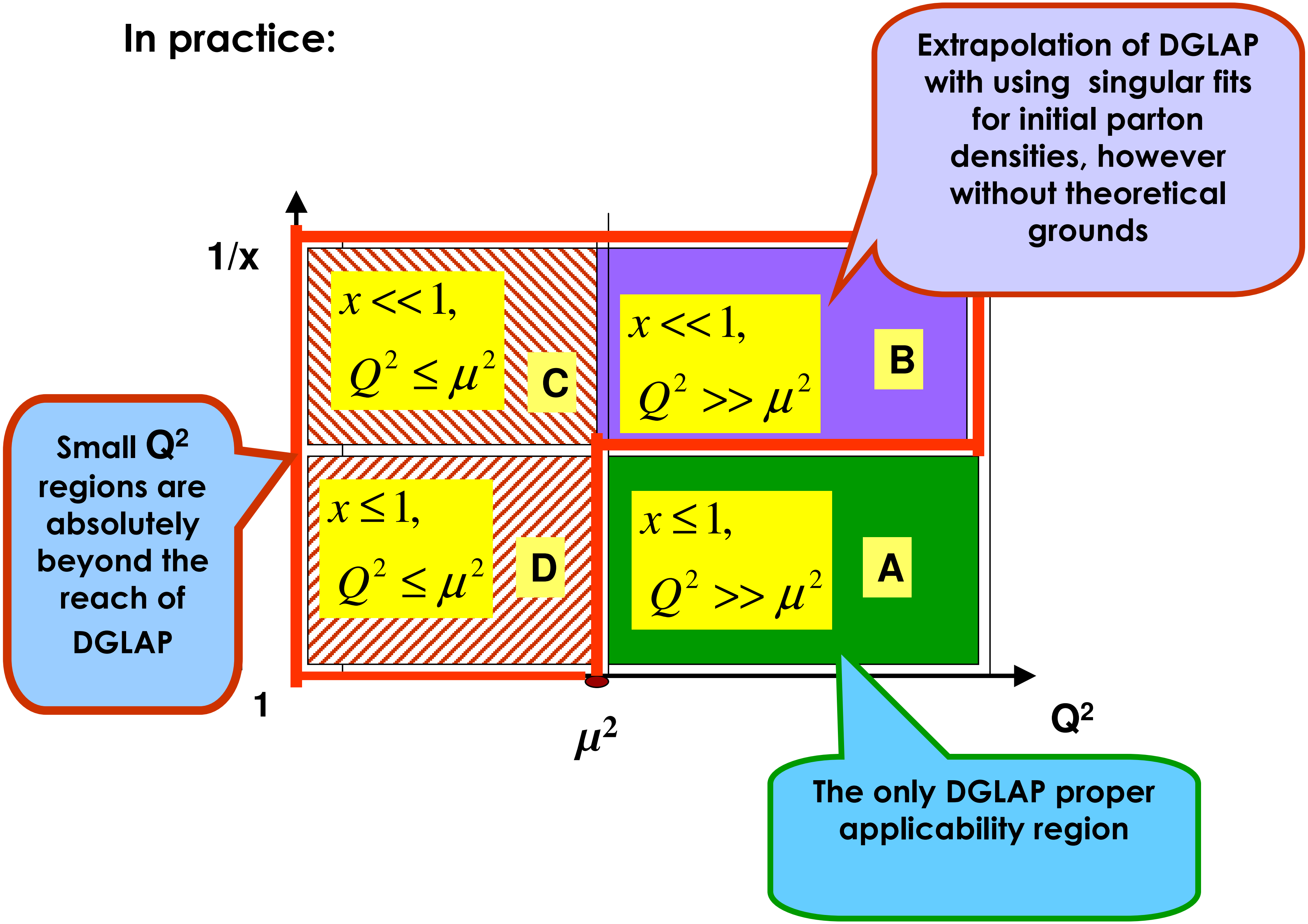
# $g_1$ at small $Q^2$



## Therefore from theoretical grounds:



In practice:



Extrapolation of DGLAP with using singular fits for initial parton densities, however without theoretical grounds
1/x
$x << 1,$
$Q^2 \leq \mu^2$
C
$x << 1,$
$Q^2 >> \mu^2$
B
Small $Q^2$ regions are absolutely beyond the reach of DGLAP
$x \leq 1,$
$Q^2 \leq \mu^2$
D
$x \leq 1,$
$Q^2 >> \mu^2$
A
1
$\mu^2$
$Q^2$
The only DGLAP proper applicability region

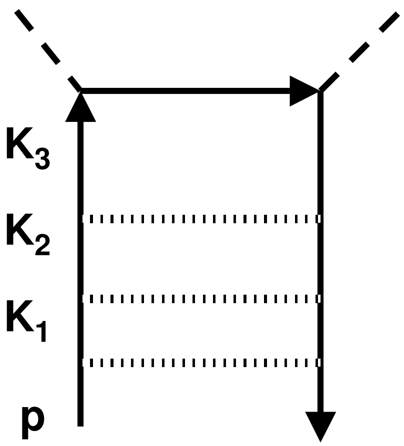
Therefore, DGLAP can be used in Region A only and the problem is how to describe $g_1$ in Regions B,C,D

## Description of $g_1$ in Region B: small x and large $Q^2$

Problems have to be solved:

- Accounting for leading logarithms of x
- Treatment of the QCD coupling at small x

DGLAP cannot do total resummation of logs of x because of the DGLAP-ordering – KEYSTONE of DGLAP




DGLAP –ordering:

$$\mu^2 < k_{1\perp}^2 < k_{2\perp}^2 < k_{3\perp}^2 < Q^2$$

good approximation for large x when logs of x can be neglected. At x << 1 the ordering has to be lifted. It makes possible to account for leading logs of x

**DL contributions**

**SL contributions**

$$(\alpha_s \ln^2(1/x))^k,$$

$$(\alpha_s \ln (1/x) \ln(Q^2/\mu^2))^k \qquad (\alpha_s \ln(1/x))^k,$$

$$k = 1,2..\infty$$

**DGLAP is free of infrared divergences:**

$$\int_{\mu^2}^{Q^2} \frac{dk_{1\perp}^2}{k_{1\perp}^2}, \ldots \int_{k_{n-1\perp}^2}^{Q^2} \frac{dk_{n\perp}^2}{k_{n\perp}^2}$$

## Lifting DGLAP –ordering causes infrared divergences in gluon ladders and non-ladder quark and gluon graphs:

**Sudakov parametriization** $k_n = \alpha_n (q + xp) + \beta_n p + k_{n\perp}$

**DGLAP ordering** $\mu^2 < \mathrm{k}_{1\perp}^2 < \mathrm{k}_{2\perp}^2 < \mathrm{k}_{3\perp}^2 < Q^2$

**Should be changed for the new ordering:**

$$1 > \beta_1 > \beta_2 > \ldots > \beta_n \qquad \mu^2 < \frac{\mathrm{k}_{1\perp}^2}{\beta_1} < \frac{\mathrm{k}_{2\perp}^2}{\beta_2} < \ldots . < \frac{\mathrm{k}_{n\perp}^2}{\beta_n} < W$$

⇓

$$\int_{\mu^2}^{W} \frac{dk_{1\perp}^2}{k_{1\perp}^2}, \ldots \int_{\mu^2}^{W} \frac{dk_{n\perp}^2}{k_{n\perp}^2} \quad \text{with} \quad W = 2pq$$

NEXT IMPORTANT STEP:

What is appropriate parameterization of $\alpha_s$ at small x ?

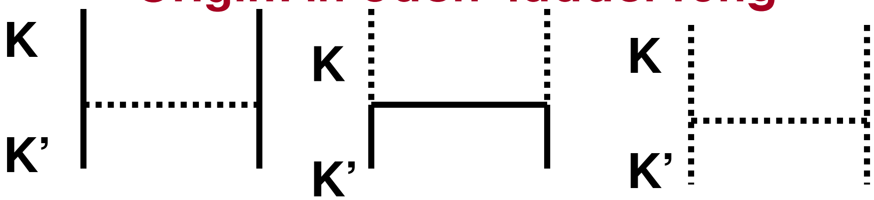
Standard parameterization $\alpha_s = \alpha_s(Q^2)$

DGLAP-parameterization

Arguments in favor of the $Q^2$- parameterization:

Amati-Bassetto-Ciafaloni-Marchesini - Veneziano; Dokshitzer-Shirkov

Origin: in each ladder rung




$$\alpha_s = \alpha_s(k_\perp^2)$$

DGLAP-parameterization

$$\alpha_s = \alpha_s(Q^2)$$

However, such a parameterization is good for large x only. At small x :

$$\alpha_s = \alpha_s((k-k')^2) \neq \alpha_s(k_\perp^2)$$

Ermolaev-Greco-Troyan

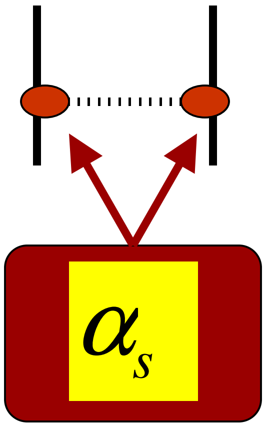
time-like argument

Participates in the Mellin transform

**At large x**

$$\alpha_s((k-k')^2) \approx \alpha_s((k_\perp^2 + k_\perp'^2)/x) \approx \alpha_s(k_\perp^2)$$

When DGLAP- ordering is used and x ~1




virtualities of all external lines are small, no $Q^2$ at all

$$\alpha_s(s) = \frac{1}{b\ln(-s/\Lambda_{QCD}^2)} =$$

$$\frac{1}{b[\ln(s/\Lambda_{QCD}^2) - i\pi]} = \frac{\ln(s/\Lambda_{QCD}^2) + i\pi}{b[\ln^2(s/\Lambda_{QCD}^2) + \pi^2]}$$

The coupling participates in the Mellin transform

$$M_B = \alpha_s(s)\frac{s}{s-\mu^2+i\varepsilon} \to \frac{A(\omega)}{\omega}$$

where

$$\alpha_s(s) \to A(\omega) = \frac{1}{b}\left[\frac{\eta}{\eta^2+\pi^2} - \int_0^\infty d\rho \frac{\exp(-\omega\rho)}{(\rho+\eta)^2+\pi^2}\right]$$

with

$$\eta = \ln(\mu^2/\Lambda^2_{QCD})$$

It is valid when $\mu^2 > \Lambda^2_{QCD}$

This restriction guarantees the applicability of Pert QCD

## Expression for the non-singlet $g_1$ at small x and large $Q^2$: $Q^2 >> 1$ GeV$^2$

Initial quark density

$$g_1^{NS} = \frac{e_q^2}{2}\int_{-i\infty}^{i\infty}\frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega}\left(\frac{\omega}{\omega - H(\omega)}\right)\delta q(\omega)\left(\frac{Q^2}{\mu^2}\right)^{H(\omega)}$$

Coefficient function

Anomalous dimension

**New coefficient function and anomalous dimension sum up leading logarithms to all orders in $\alpha_s$**

## Compare our non-singlet anomalous dimension to the LO DGLAP one:

**expand C and H into series in** $1/\omega$

$$H = \frac{A(\omega)C_F}{2\pi}\left[\frac{1}{\omega}+\frac{1}{2}\right]+\ldots$$

**coincide, save the treatment of** $\alpha_s$

$$\gamma_{NS}^{\text{LO DGLAP}}=\frac{\alpha_s(Q^2)C_F}{2\pi}\left[\frac{1}{n(n+1)}+\frac{3}{2}-S_2(n)\right]\approx\frac{\alpha_s(Q^2)C_F}{2\pi}\left[\frac{1}{n}+\frac{1}{2}+O(n)\right]$$

**when n < 1**

**where**

$$S_k(n)=\sum_{j=1}^{n}\frac{1}{j^k}$$

**small/large x** ⟷ **small/large n**

## Compare our coefficient function and the NLO DGLAP one

$$C = \frac{\omega}{\omega - H(\omega)} = 1 + \frac{A(\omega) C_F}{2\pi}\left[\frac{1}{\omega^2} + \frac{1}{2\omega}\right] + ....$$

coincide, save the treatment of $\alpha_s$

LO

NLO

$$C_{NS}^{\mathrm{DGLAP}} = 1 + \frac{\alpha_s(Q^2) C_F}{2\pi}\left[\frac{1}{n^2} + \frac{1}{2n} + \frac{1}{2n+1} - \frac{9}{2} + \left(\frac{3}{2} - \frac{1}{n(1+n)}\right) S_1(n) + S_1^2(n) - S_2(n)\right]$$

when n < 1

$$\approx \frac{\alpha_s(Q^2) C_F}{2\pi}\left[\frac{1}{n^2} + \frac{1}{2n} + O(n)\right]$$

**Expression for the singlet $g_1$ at small x and large $Q^2$:**

$$g_1^S = \frac{<e_q^2>}{2}\int \frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega}$$

$$\left[\left(C_q^{(+)}\delta q + C_q^{(+)}\delta g\right)\left(\frac{Q^2}{\mu^2}\right)^{\Omega^{(+)}} + \left(C_q^{(-)}\delta q + C_q^{(-)}\delta g\right)\left(\frac{Q^2}{\mu^2}\right)^{\Omega^{(-)}}\right]$$

Initial quark density

Initial gluon density

Initial quark density

Initial gluon density

$$Q^2 > \mu^2; \ \mu \approx 5 \text{ GeV}$$

## Numerical comparison of our results to DGLAP

Comparison depends on the assumed shape of initial parton densities.

The simplest option: use the bare quark input

$$\delta q(x) = \delta(x) \quad \Longleftrightarrow \quad \delta q(\omega) = 1$$

in x- space | in Mellin space

Numerical comparison shows that the impact of the total resummation of logs of x becomes quite sizable at $x = 0.05$ approx.

PUZZLE: DGLAP should have Failed at $x < 0.05$.
However, it does not take place.

In order to understand what could be the reason for success of DGLAP at small x, let us consider in more detail

**standard fits for initial parton densities.**

$$\delta q(x) = N\, x^{-\alpha}[(1+\gamma x^{\delta})(1-x)^{\beta}]$$

Altarelli-Ball-Forte-Ridolfi

normalization

singular factor

regular factors

parameters $\alpha \approx 0.58,\ \beta \approx 2.7, \gamma \approx 34.3,\ \delta \approx 0.75$

are fixed from fitting experimental data at large x

In the Mellin space this fit is

$$\delta q(\omega) = N[(\omega-\alpha)^{-1} + \sum_{k=1}^{\infty} c_k((\omega+k-\alpha)^{-1} + \gamma(\omega+k+1-\alpha)^{-1})]$$

**Leading pole**
$\alpha$=0.58 >0

**Non-leading poles**
-k +$\alpha$<0

the small-x DGLAP asymptotics of $g_1$ is (inessential factors dropped )

$$g_1^{DGLAP} \sim (1/x)^{\alpha}$$

**phenomenology**

Comparison of it to our asymptotics

$$g_1 \sim (1/x)^{\Delta_{NS}}$$

**calculations**

shows that the singular factor in the DGLAP fit mimics the total resummation of ln(1/x) . However, the value $\alpha$ = 0.58 sizably differs from our non-singlet intercept =0.42

Common opinion: fits for $\delta$**q** are defined at large x,
then convoluting them with coefficient functions weakens the singularity

$$C(x, y) \otimes \delta q(y) = \Delta q(x)$$

initial — x-evolved

Obviously, it is not true:
They both are singular equally

Structure of DGLAP fit once again:

$$\delta q(x) = N\, x^{-\alpha}[(1+\gamma x^{\delta})(1-x)^{\beta}]$$

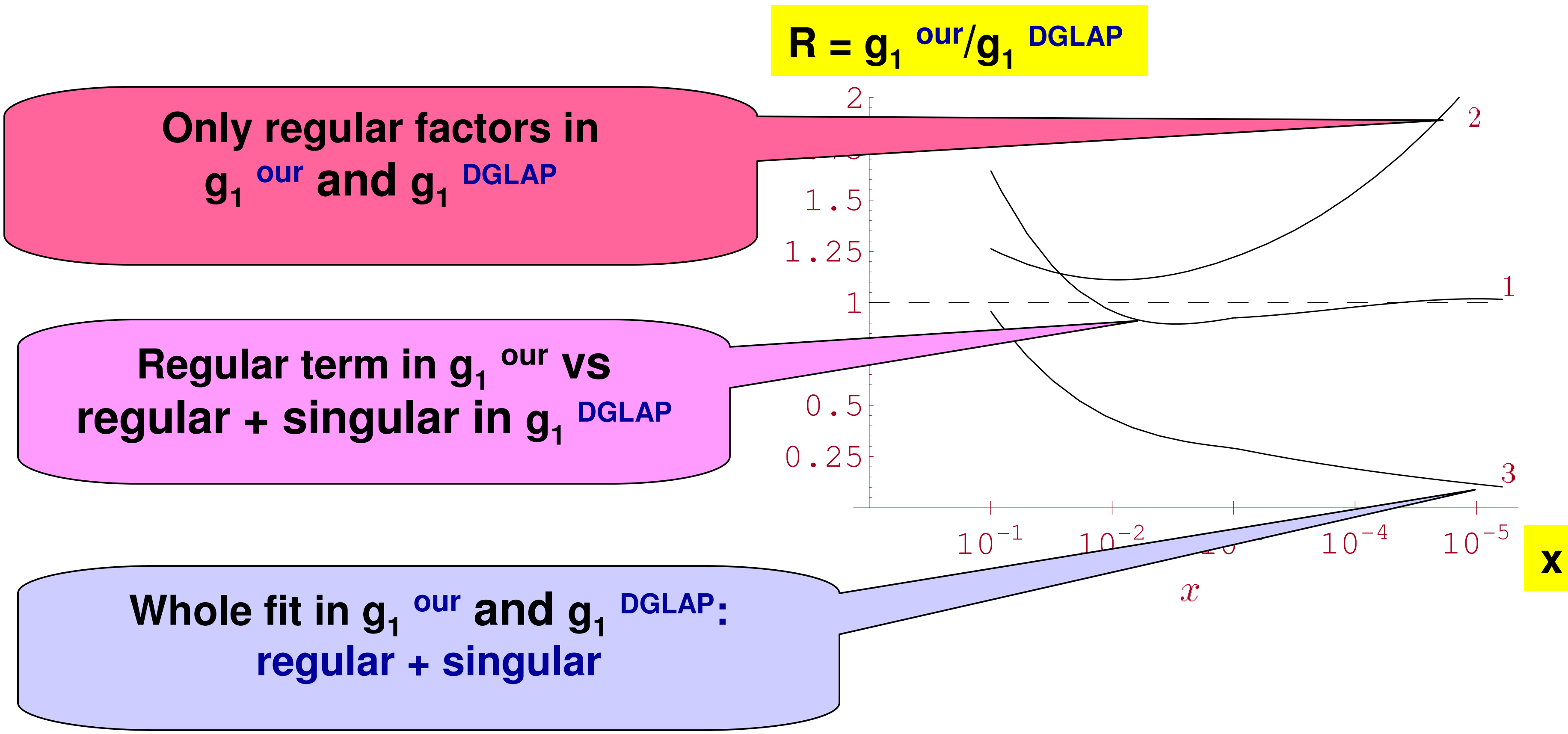
Can be dropped when ln(x) are resummed

x-dependence is weak at x<<1 and can be dropped

Therefore at x << 1

$$\delta q(x) \approx \mathrm{N}(1 + \mathrm{a}x)$$

## Numerical comparison of DGLAP with our approach at small but finite x, using the same DGLAP fit for initial quark density.



## Description of $g_1$ in Region C: small $Q^2$ and small x:

**Generalization of our previous results through the shift**

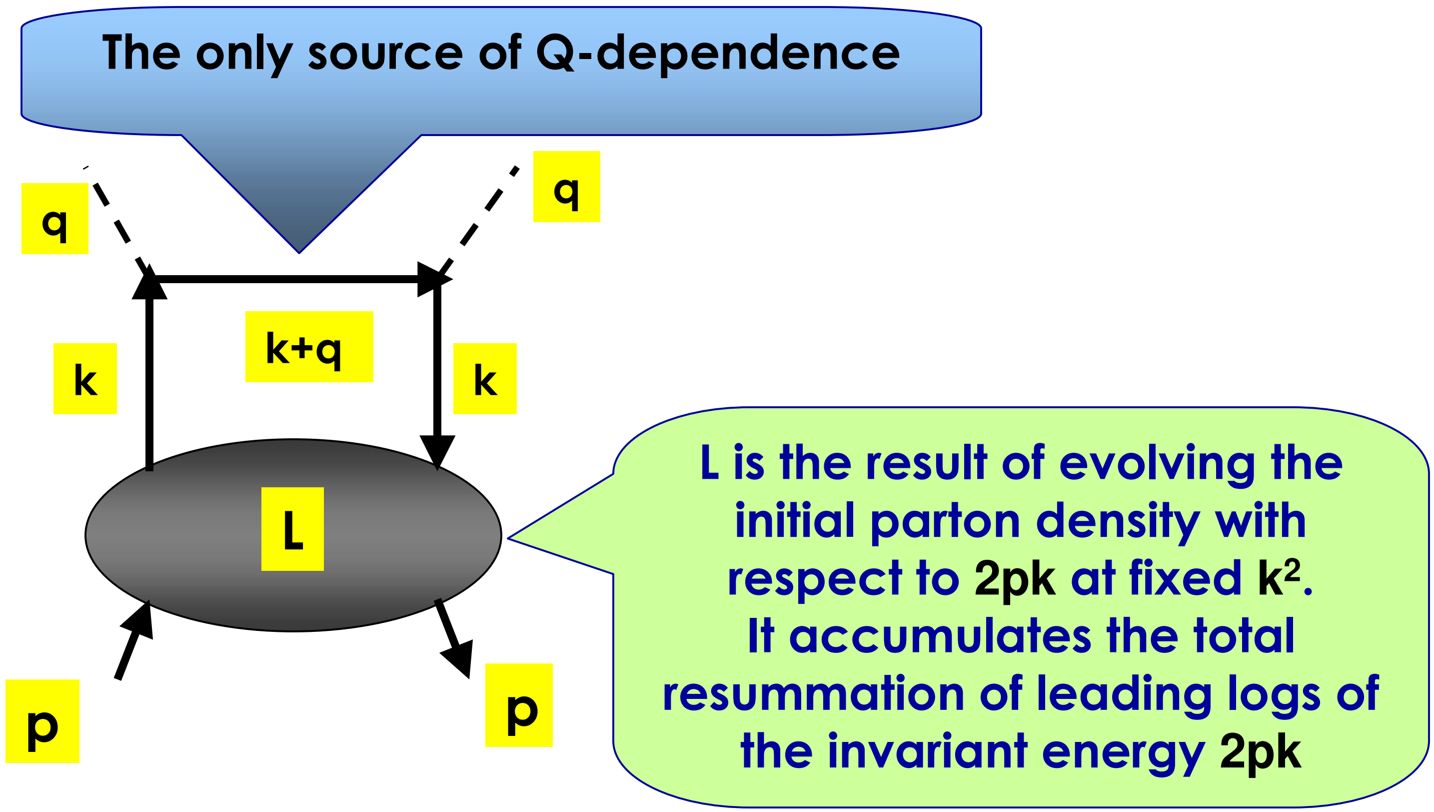
$$Q^2 \rightarrow Q^2 + \mu^2 \quad \Rightarrow \quad x \rightarrow \bar{x} = (Q^2 + \mu^2)/2pq = x + z$$

Infrared cut-off

**Similar shifts have been used for DIS structure functions by many authors, however from phenomenological considerations. We do It from analysis of the involved Feynman graphs**

## Obviously, $g_1$ obeys the Bete-Salpeter equation:

$g_1=$




$$g_1 = g_1^{Born} + \int \frac{d^4k \, k_\perp^2}{(k^2 - m_q^2)^2} \delta(k^2 + 2qk - (Q^2 + \mu^2)) L(2pk, k^2, \mu^2)$$

introducing the IR cut-off $\mu >> m_q$ into singular (vertical)

propagators and using the Sudakov parameterization

$$k = \alpha q + (\beta + x\alpha) p + k_{\perp} \approx \alpha q + \beta p + k_{\perp}$$

we obtain

$$g_1 = g_1^{Born} + \int_{Q^2}^{w} \frac{dk_{\perp}^2}{k_{\perp}^2 + \mu^2} L(w, k_{\perp}^2 + \mu^2) =$$

$$g_1^{Born} + \int_{Q^2 + \mu^2}^{w + \mu^2} \frac{dt}{t} L(w, t)$$

proves the shift

**It leads to new expressions:** **non-singlet $g_1$ at small x and arbitrary $Q^2$**

$$z = \frac{\mu^2}{2pq} >> x = \frac{Q^2}{2pq}$$

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left( \frac{1}{z + x} \right)^{\omega} \left( \frac{\omega}{\omega - H(\omega)} \right) \delta q(\omega) \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{H(\omega)}$$

x -dependence

Anomalous dimension

$Q^2$ –dependence

Coefficient function

Initial quark density

## Singlet $g_1$ at small x and arbitrary $Q^2$

$$z = \frac{\mu^2}{2pq}, \quad x = \frac{Q^2}{2pq}$$

$$g_1^S = \frac{<e_q^2>}{2} \int \frac{d\omega}{2\pi i} \left( \frac{1}{z+x} \right)^{\omega} \left[ C_q \delta q + C_g \delta g \right]$$

$$C_g = C_g^{(+)} \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(+)}} + C_g^{(-)} \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(-)}}$$

$$C_q = C_q^{(+)} \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(+)}} + C_q^{(-)} \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(-)}}$$

where $\Omega^{(\pm)}, \ C_q^{(\pm)}, C_g^{(\pm)}$

contain total resummation of ln(1/x)

## Unified description of $g_1$ in Regions A&B:
## large $Q^2$ and arbitrary x:

| DGLAP | our approach |
|---|---|
| Good at large x because includes exact two-loop calculations but bad at small x as it lacks the total resummaion of ln(x) | Good at small x , includes the total resummaion of ln(x) but bad at large x because neglects some contributions essential in this region |

**WAY OUT – interpolation expressions combining our approach and DGLAP**

1. Expand our formulae for coefficient functions and anomalous dimensions into series in the QCD coupling
2. Replace the first- and second- loop terms of the expansion by corresponding DGLAP –expressions

## Non-singlet $g_1$: Our expressions

$$H_{LL}(\omega) = (1/2)[\omega - (\omega^2 - B(\omega))]^{1/2} \quad C_{LL}(\omega) = \omega/(\omega - H(\omega))$$

anomalous dimension

coefficient function

First tems of their expansions into the perturbation series

$$H_1 = \frac{A(\omega)C_F}{2\pi}\left[\frac{1}{\omega} + \frac{1}{2}\right] \quad C_1 = \frac{A(\omega)C_F}{2\pi}\left[\frac{1}{\omega^2} + \frac{1}{2\omega}\right]$$

New formulae combine Resummation and DGLAP:

$$H_C = H_{LL} - H_1 + H_{LO\,DGLAP} \quad C_C = C_{LL} - C_1 + C_{LO\,DGLAP}$$

New, combined or “synthetic”, formulae for the singlet anomalous dimensions and coefficient functions are written quite similarly

## Technology of getting universal description of $g_1$:

**Step 1:**
Resummation of leading $\ln(1/x)$ and $\ln(Q^2)$

→ $g_1$ in Region B: large $Q^2$ and small x

**Step 2:**
Combining above results and DGLAP

→ $g_1$ in Region A&B: large $Q^2$ and arbitrary x

**Step 3:**
Shift $Q^2 \rightarrow Q^2 + \mu^2$

→ $g_1$ in Region C&D: small $Q^2$ and arbitrary x

Thus, we arrive at universal and model-independent description of $g_1$ at arbitrary $Q^2$ and x without singular fits:

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left( \frac{1}{z + x} \right)^{\omega} C_C(\omega)\, \delta q(\omega) \left( \frac{\mu^2 + Q^2}{\mu^2} \right)^{H_C(\omega)}$$

$$z = \frac{\mu^2}{2pq}, \quad x = \frac{Q^2}{2pq}$$

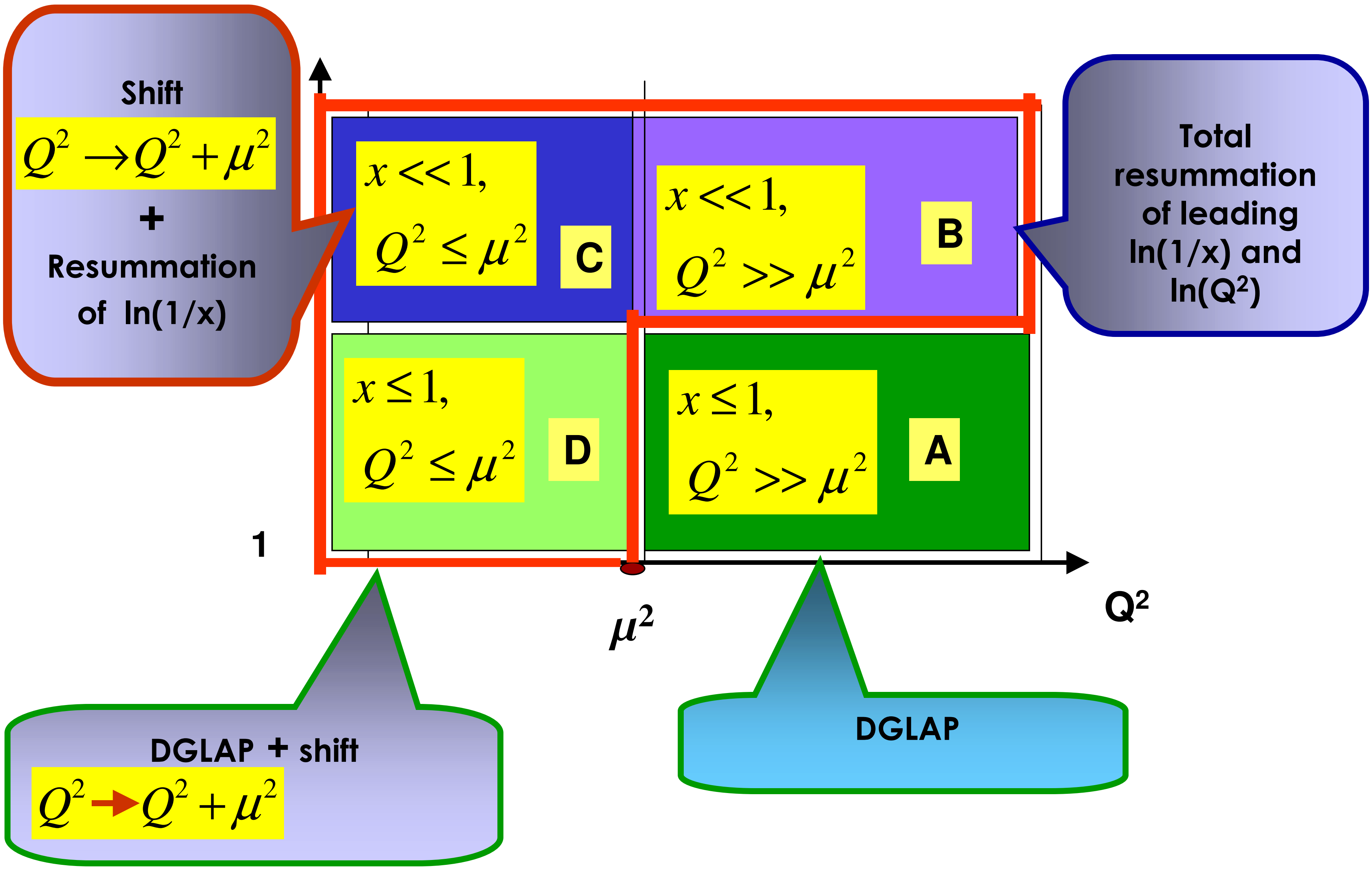
Combined coefficient function

Non-singular quark density

Combined anomalous dimension

expression for the singlet $g_1$ is written quite similarly

# Main impact on $g_1$ in Regions A,B,C,D comes from:



Recent applications of our approach to:

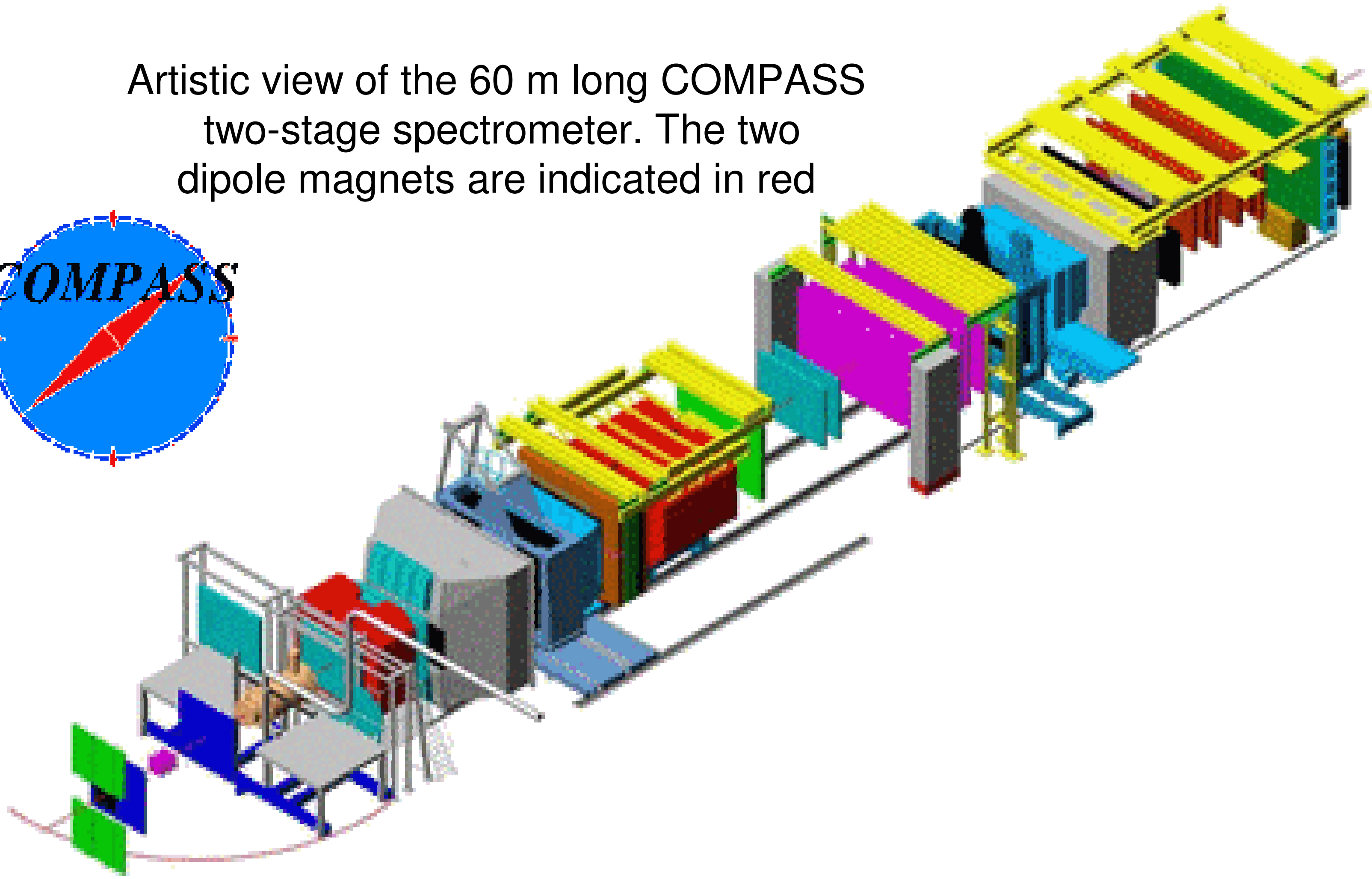
1. COMPASS results

2. Power $Q^2$-corrections

Taken from wwwcompass.cern.ch

# COMPASS

## COmmon Muon Proton Apparatus for Structure and Spectroscopy



Artistic view of the 60 m long COMPASS two-stage spectrometer. The two dipole magnets are indicated in red

**COMPASS**: $10^{-1}$ GeV$^2$ < $Q^2$ < 3 GeV$^2$ → DGLAP cannot be used:

$$\ln\left[\frac{\ln\left(Q^2/\Lambda^2\right)}{\ln\left(\mu^2/\Lambda^2\right)}\right] > 1 \Rightarrow \quad Q^2 >> \mu^2$$

**Our approach is not sensitive to values of Q2, so we can use it**

**Prediction 1:** very weak dependence $g_1$ on x at the COMPASS range of $Q^2$ even at very small x (x ~$10^{-3}$)

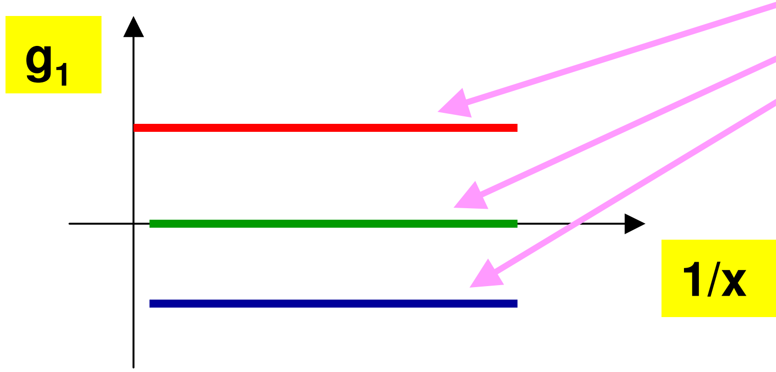
when $Q^2 << \mu^2$

$$x << z \Rightarrow g_1(x+z) \approx g_1(z) + x d g_1(z)/dz + \ldots$$

$$z = \mu^2/(2pq)$$

$$g_1(z) = \left(\frac{<e_q^2>}{2}\right)\int_{-i\infty}^{i\infty}\frac{d\omega}{2\pi i}\left(\frac{1}{z}\right)^{\omega}\left[C_q(\omega)\delta q + C_g(\omega)\delta g\right]$$

so $Q^2$- dependences is flat, even for $x<<1$.




Location of the line is determined by the z-dependence

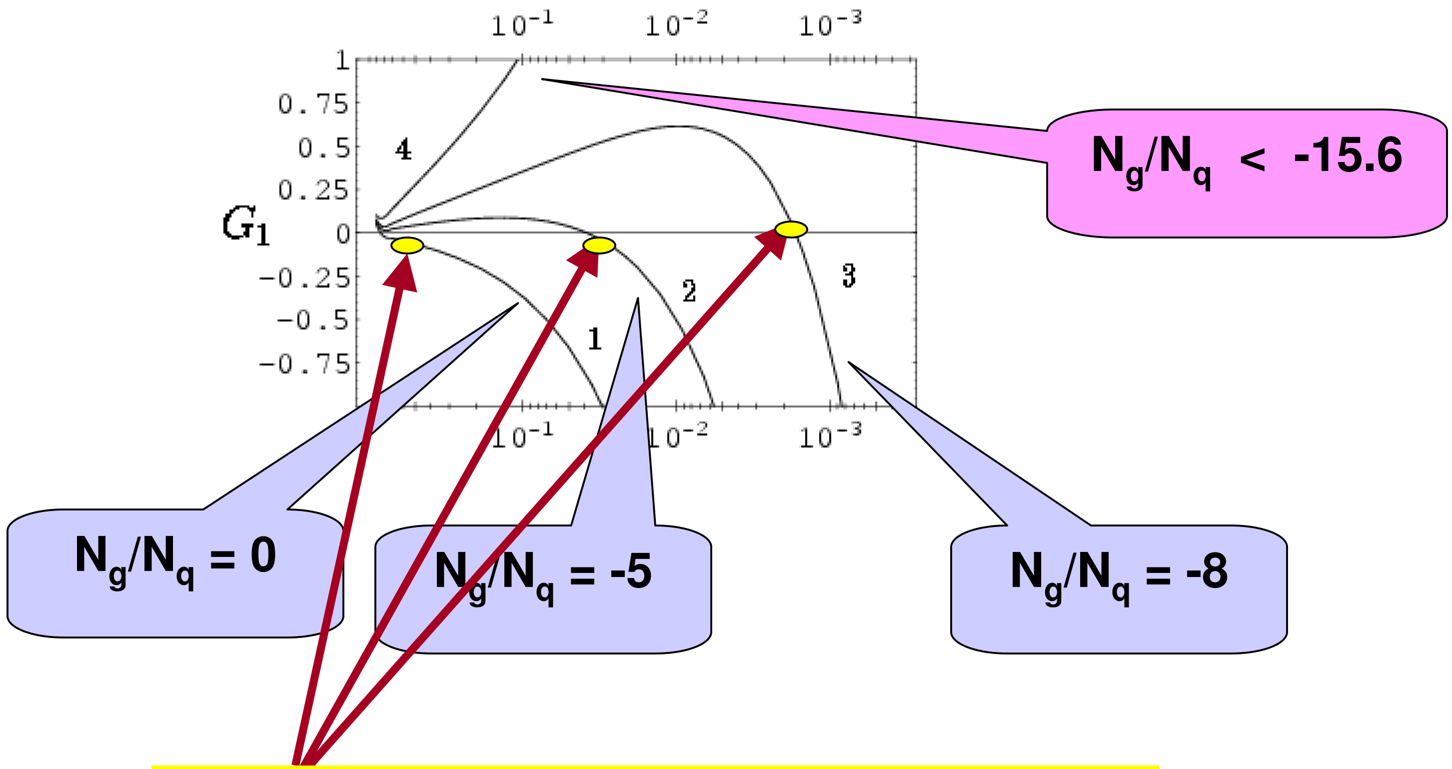
**Prediction 2:**

**Instead of studying the x-dependence, it would be much more interesting to study the w-dependence, w=2pq and get the gluon initial density from there**

$$g_1(z) = \left( \frac{< e_q^2 >}{2} \right) \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left( \frac{1}{z} \right)^{\omega} \left[ C_q(\omega)\delta q + C_g(\omega)\delta g \right]$$

Assuming $\delta q \approx N_q, \delta g \approx N_g,$ and introducing $g_1 = (e_q^2/2) N_q G_1,$

We perform numerical calculations of $G_1$




turning points for different values of $N_g/N_q$

**Position of the turning point is sensitive to $N_g/N_q$ , so the experimental detection of it will allow to estimate ratio Ng / Nq**

**Current status of our predictions:**

**Prediction 1 – confirmed by COMPASS**

**Prediction 2- is going to be checked soon by COMPASS**

# Power Corrections to non-singlet $g_1$

Leading twist contribution

mass scale: $Q^2 > M^2$

$$g_1(x,Q^2) = g_1^{LT}(x,Q^2)\left[1 + \sum_k C_k \left(\frac{M^2}{Q^2}\right)^k\right]$$

Power corrections

PC are supposed to come from higher twists.
No satisfactory theory
is known for the higher twists

Standard way of obtaining PC from experimental data at small x: Leader-Stamenov- Sidorov

Compare experimental data to predictions of the Standard Approach and assign the discrepancy to the impact of PC

$$g_1^{LT} = g_1^{DGLAP}$$

Counter-argument:

1. DGLAP, the main ingredient of SA, is unreliable at small x, so comparing experiment to it is not productive: it proves nothing
2. SA cannot explain why PC appear at $Q^2 > 1$ GeV$^2$ only and predict what happens at smaller $Q^2$

Our approach can do it:

$$g_1^{NS} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{\mathrm{w}}{\mu^2+Q^2}\right)^{\omega} \mathrm{C}(\omega)\,\delta\mathrm{q}(\omega)\left(\frac{\mu^2+Q^2}{\mu^2}\right)^{H(\omega)}$$

where w = 2pq and $Q^2$ can be large or small, $\mu$ = 1 GeV

$\mu$ **=1 GeV**, so when $Q^2 < 1$ GeV$^2$, expansion into power series is:

$$g_1^{NS} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{w}{\mu^2}\right)^{\omega} C(\omega)\,\delta q(\omega)\left[1+\sum_{k=1} T_k(\omega)\left(\frac{Q^2}{\mu^2}\right)^{k}\right]$$

Power corrections

Leading contribution for $g_1^{NS}$ does not depend on $Q^2$

**At Q² > 1 GeV² expansion into series is different:**

$$g_1^{NS} =$$

$$\frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{\mathrm{w}}{Q^2}\right)^{\omega} \mathrm{C}(\omega)\,\delta\mathrm{q}(\omega)\left(\frac{\mu^2}{Q^2}\right)^{H(\omega)}$$

$$\left[1+\sum_{k=1} T_k(\omega)\left(\frac{\mu^2}{Q^2}\right)^{k}\right]$$

Conventionally looking Power Corrections

Leading contribution for $g_1^{NS}$

**These Power Corrections have perturbative origin and should be accounted in the first place. Only AFTER THAT one can reliable estimate a genuine impact of higher twist contributions**

# Conclusion

DGLAP is theoretically based for describing $g_1$ only in Region A: large x and large $Q^2$

Extrapolating DGLAP to Regions C,D (small $Q^2$) is impossible because there is no evolution in ln($Q^{2)}$ in these Regions

Conventional extrapolating DGLAP into Region B (small x and large $Q^2$ ) has no theoretical grounds and leads to various misconceptions

## LIST OF MOST SERIOUS MISCONCEPTIONS

**Misconception:** Standard fits mimic non-perturbative (basically unknown) physics

**Actually**: the singular factor in the fits mimic the lack of total resummation of ln(1/x) in DGLAP. Their only role is to give fast growth to $g_1$ at small x. They should be dropped whenResummation is accounted for and therefore the fits are becoming simpler

**Misconception:** Total resummation of logs of x brings only small impact on the small-x behavior of $g_1$
**Actually**: It happens when both Resummation and Standard singular fits are used together. In this
In this case the same logs of x are accounted twice: first implicitly through the fits and secondly explicitly trough Resummation. Besides, this approach predicts incorrect intercepts

**Misconception:** Conventional $Q^2$-corrections are believed to correspond to non-perturbative QCD, so they are attributed to higher twists.
**Actually**: At least a part of these corrections, if not all of them, have the perturbative origin. Impact of higher twists should be determined only after accounting for the perturbative $Q^2$-corrections

The appropriate way to consider $g_1$ at small x (Regions B,C) is total resummation of leading logs of x and the shift $Q^2 \rightarrow Q^2 + \mu^2$

Combining those expressions and DGLAP formulae for anomalous dimensions and coefficient functions leads to universal description of $g_1$ in Regions A,B,C,D, however with much simpler fits for initial parton densities