A Lamb Shift Polarimeter for a Helium-3 Ion Beam

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Hyperfine structure of $^3He^+$

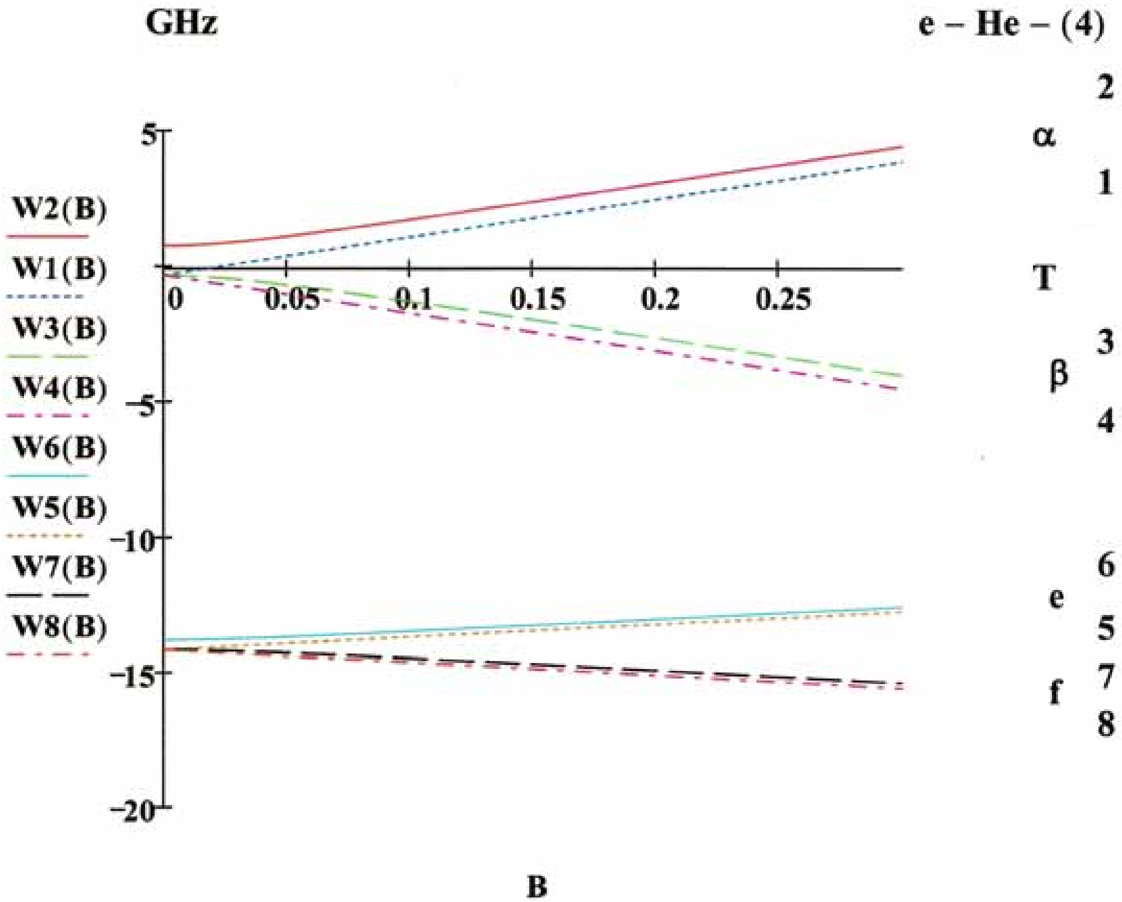
$2S_{\frac{1}{2}}$

e + He − (2)

e + He + (1)

e − He + (3)

e − He − (4)



$\Delta m := 1$ $\quad B := 0.25 \cdot T$

	ν(GHz)
(1) − −(7)	18.37
(2) − −(8)	19.30
(3) − −(5)	9.69
(4) − −(6)	9.00

$2P_{\frac{1}{2}}$

e + He − (6)

e + He + (5)

e − He + (7)

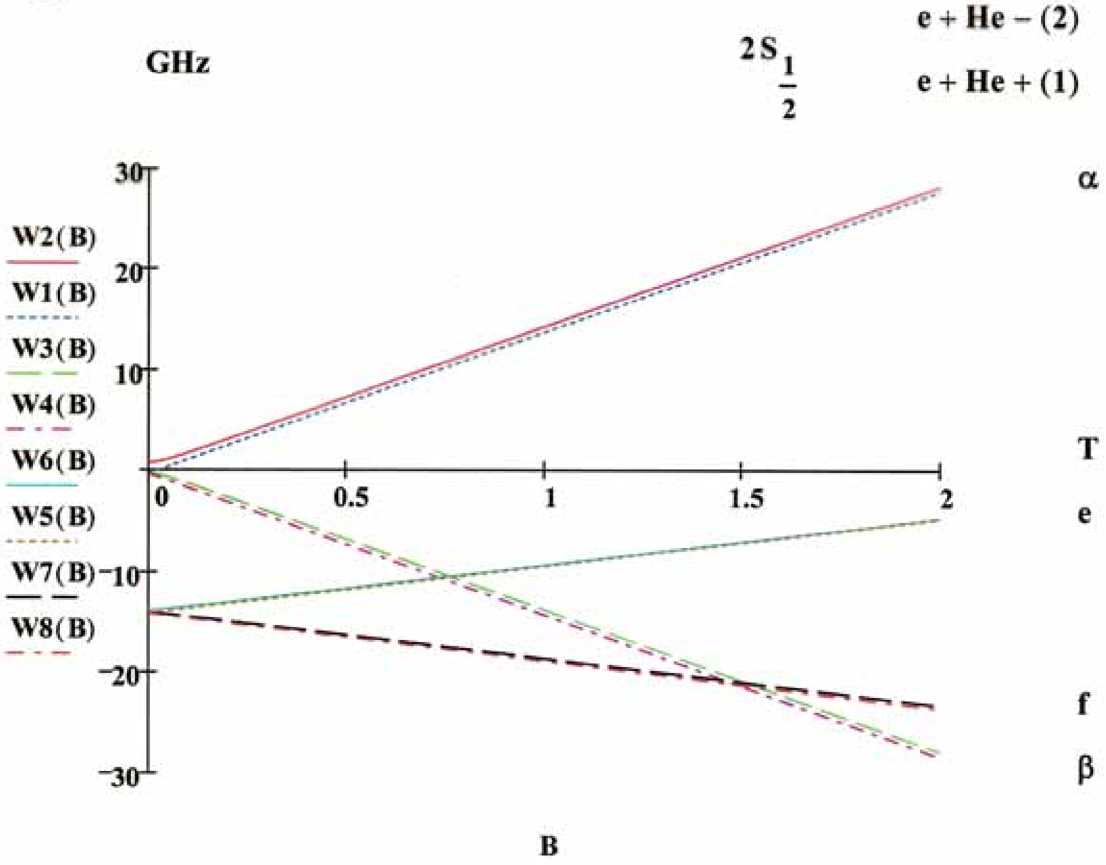
e − He − (8)

$\nu_0 := 9.35 \cdot GHz$

$\lambda := 3.2 \cdot cm$

$E := 300 \cdot \frac{V}{cm}$

Hyperfine structure of $^3He^+$



$B1 := 0.77 \cdot T$

$\alpha - - e \qquad \nu := 21 \cdot GHz \qquad \lambda := 1.4 \cdot cm$

$B2 := 1.5 \cdot T$

e + He − (6)

$2P_{\frac{1}{2}}$

e + He + (5)

e − He + (7)

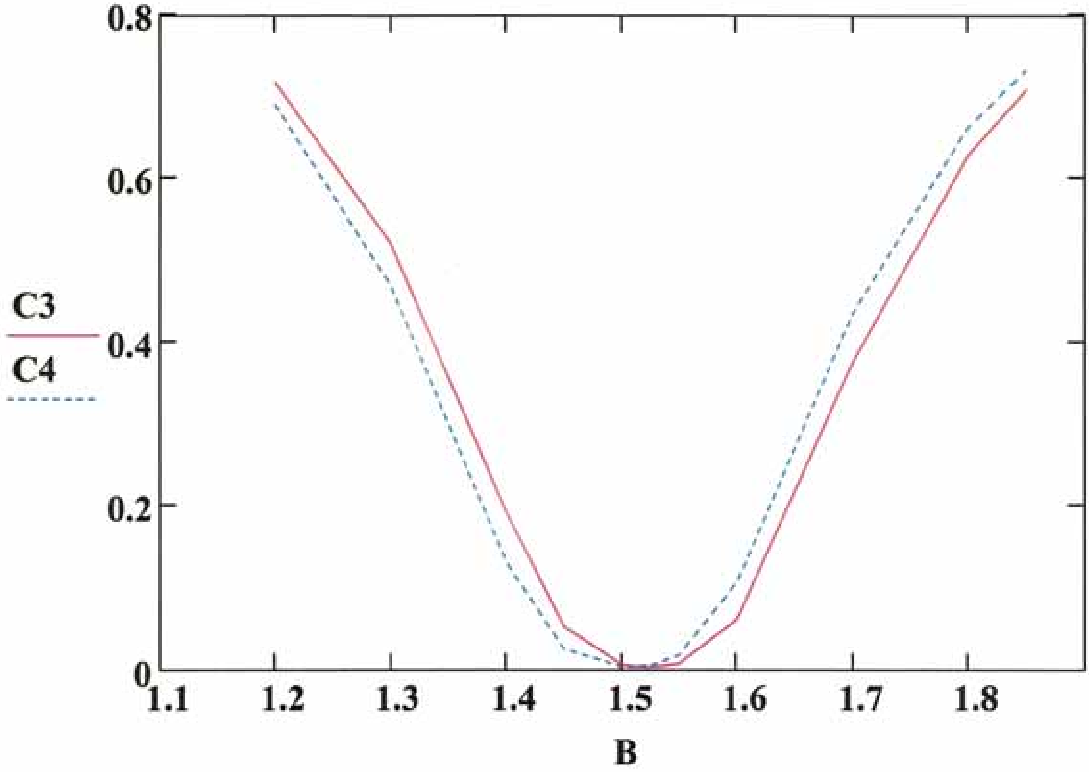
$2P_{\frac{1}{2}}$

e − He − (8)

e − He + (3)

$2S_{\frac{1}{2}}$

e − He − (4)

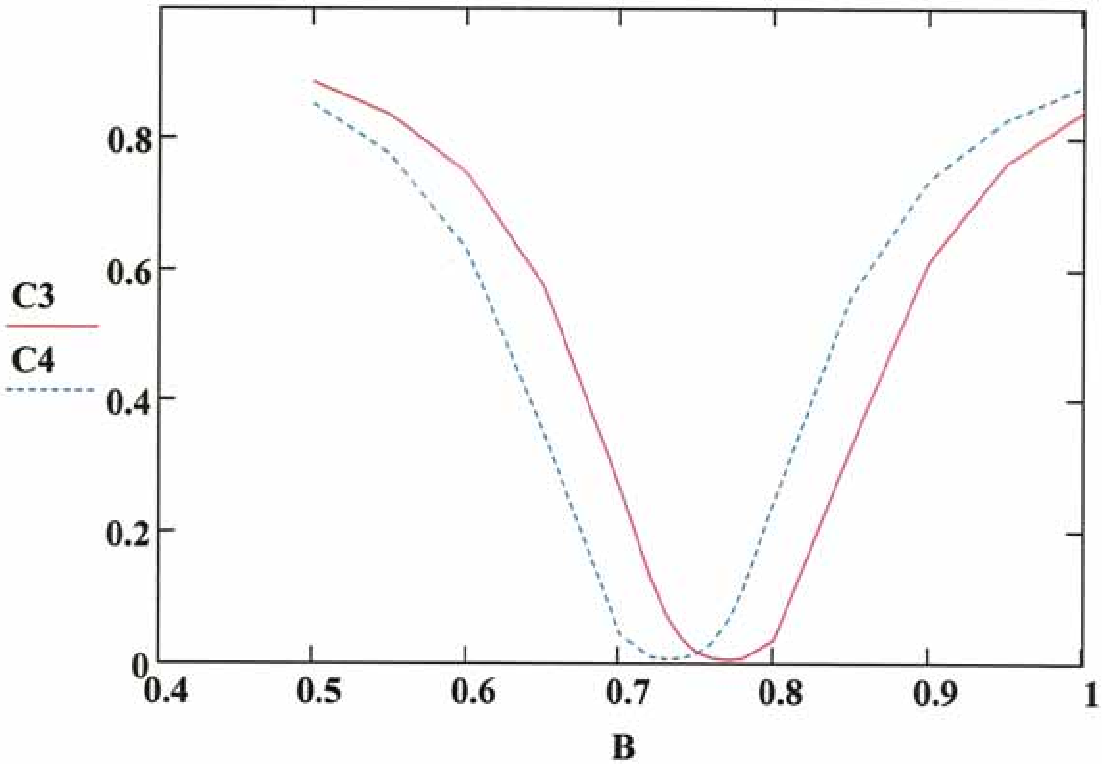


Probabilty to remain in the 2S state

$C1 := 0.996$ $\qquad$ $C2 := 0.996$

$t := 10^{-8} \cdot sec$ $\qquad$ $Ec := 150 \cdot \frac{V}{cm}$ $\qquad$ B , Tesla

$v := 1.13 \cdot 10^{8} \cdot \frac{cm}{sec}$ $\qquad$ $E := 20 \cdot kV$

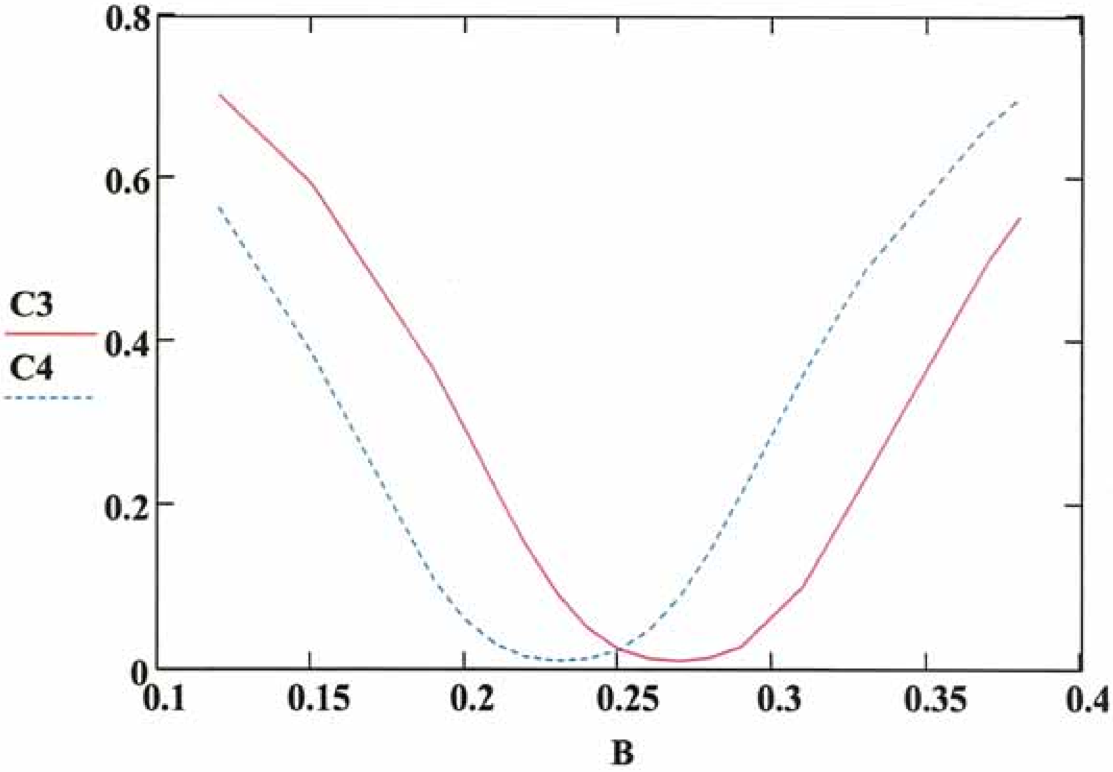


Probability to remain in the 2S state

$$C1 := 0.995 \qquad C2 := 0.995$$

$$t := 3\cdot 10^{-8}\cdot sec \qquad Ec := 90\cdot\frac{V}{cm}$$

$$v := 1.13\cdot 10^{8}\cdot\frac{cm}{sec} \qquad E := 20\cdot kV \qquad B\text{, Tesla}$$



Probability to remain in the 2S state

$C1 := 0.96$ $C2 := 0.97$

$t := 10^{-8} \cdot sec$ $Erf := 300 \cdot \frac{V}{cm}$ B, Tesla

$\nu_0 := 9.35 \cdot GHz$

In their experiment on the fine structure of hydrogen Lamb and Retherford showed that contrary to the predictions of the Dirac theory, states with the same principal quantum number, n, and the same total angular momentum quantum number, j, were not degenerate. They measured the energy difference between the $2S_{1/2}$ and $2P_{1/2}$ states (Lamb shift) and obtained the value of 1057.77 ± 0.10 MHz.

The present experimental value is 1057.8514 ± 0.0019 MHz and theoretical values are 1057.910 ± 0.010 MHz and 1057.864 ± 0.014 MHz.

Lamb and Skinner also measured $2^2S_{1/2}$–$2^2P_{1/2}$ level shift in ionized helium-4 and obtained the value 14020 ± 100 MHz. The present experimental value is 14046 ± 12 MHz. The stated uncertainty is equal to three times the standard deviation plus an estimated 3 MHz for the uncertainty in the correction for systematic effects. The theoretical value is 14043.2 ± 3.0 MHz.

The method is based on the relation between the nuclear polarization of $^3He^{++}$ ions and the populations of the hyperfine levels of the $^3He^+(2S)$ ions, produced in the electron capture process

$$^3\mathrm{He}^{++} + \mathrm{X} \rightarrow {}^3\mathrm{He}^{+}(n, l, m) + (\mathrm{X}')^{+}.$$

between the incident ions and target gas atoms or molecules X.

(n, l, m) denote the principal, orbital angular momentum and magnetic quantum number of a hydrogen-like atom. Subsequent radiative decays of the initial states lead to a mixture of the desired metastable $^3He^+(2S)$ ions and the ground state ions $^3He^+(1S)$.

The cross sections for the charge-transfer processes of this type in He, Ar, Kr, H_2, N_2 and O_2 were measured by Shah and Gilbogy (1974) in an energy range of 10-60 keV. At impact energies of 20-30 keV the maximum fractional yield of $^3He^+(2S)$ ions was 2.5%.

The populations of the states of the $^3\mathrm{He}^+(2\mathrm{S})$, produced in the capture of unpolarized electrons by the $^3\mathrm{He}^{++}$ ions with polarization P (a sudden process)

$$\phi^+_{\mathrm{He}}\phi^+_{\mathrm{e}} \quad \text{population} \ \frac{1+P}{4}, \quad (1)$$

$$\phi^-_{\mathrm{He}}\phi^+_{\mathrm{e}} \quad \text{population} \ \frac{1-P}{4}, \quad (2)$$

$$\phi^+_{\mathrm{He}}\phi^-_{\mathrm{e}} \quad \text{population} \ \frac{1+P}{4}, \quad (3)$$

$$\phi^-_{\mathrm{He}}\phi^-_{\mathrm{e}} \quad \text{population} \ \frac{1-P}{4}. \quad (4)$$

These states are not the eigenfunctions of a time-independent Hamiltonian:

$$\hat{H} = -\mu_{\mathrm{e}}\vec{B}\vec{\sigma}_{\mathrm{e}} - \mu_{\mathrm{He}}\vec{B}\vec{\sigma}_{\mathrm{He}} + \frac{1}{4}\Delta W\vec{\sigma}_{\mathrm{e}}\vec{\sigma}_{\mathrm{He}}.$$

The states evolve in time

$$(1) \ \ \psi(\mathrm{F}=1,\mathrm{m}=1)\exp[-i\omega(1,1)t],$$

$$(2) \ \ \sin\beta\psi(1,0)\exp[-i\omega(1,0)t] - \cos\beta\psi(0,0)\exp[-i\omega(0,0)t],$$

$$(3) \ \ \cos\beta\psi(1,0)\exp[-i\omega(1,0)t] + \sin\beta\psi(0,0)\exp[-i\omega(0,0)t],$$

$$(4) \ \ \psi(1,-1)\exp[-i\omega(1,-1)t],$$

where

$$\psi(1,0) = \cos\beta\phi^+_{\mathrm{He}}\phi^-_{\mathrm{e}} + \sin\beta\phi^-_{\mathrm{He}}\phi^+_{\mathrm{e}},$$

$$\psi(0,0) = \sin\beta\phi^+_{\mathrm{He}}\Psi^-_{\mathrm{e}} - \cos\beta\phi^-_{\mathrm{He}}\phi^+_{\mathrm{e}}.$$

$$\sin\beta = \frac{1}{\sqrt{2}}\left(1-\frac{x}{\sqrt{1+x^2}}\right)^{1/2}, \cos\beta = \frac{1}{\sqrt{2}}\left(1+\frac{x}{\sqrt{1+x^2}}\right)^{1/2},$$

$$x = \frac{B}{B_c}, \ \ B_c = \frac{|\Delta W|}{-\mu_J/J+\mu_I/I},$$

For the $2\mathrm{S}_{1/2}$ states

$$\Delta W = -1.083355\,\mathrm{GHz}, \quad B_c = 38.61211\mathrm{mT}.$$

Zeeman effect. For $2S_{1/2}$ states:

$$W(F=1,m=1) = -\frac{\Delta W}{4} - \mu_J B - \mu_I B,$$

$$W(1,0) = -\frac{\Delta W}{4} + \frac{\Delta W}{2}\sqrt{1+x^2},$$

$$W(1,-1) = -\frac{\Delta W}{4} + \mu_J B + \mu_I B,$$

$$W(0,0) = -\frac{\Delta W}{4} - \frac{\Delta W}{2}\sqrt{1+x^2}.$$

Usually, populations of the four $2S_{1/2}$ are

$$N(1,0) = \cos^2\beta_0(1+P)/4 + \sin^2\beta_0(1-P)/4,$$

$$N(0,0) = \sin^2\beta_0(1+P)/4 + \cos^2\beta_0(1-P)/4,$$

$$N(1,1) = (1+P)/4,$$

$$N(1,-1) = (1-P)/4$$

$$N(1,0) = [1+P(\cos^2\beta_0 - \sin^2\beta_0)]/4 = \frac{1}{4}\left(1 + P\frac{x}{\sqrt{1+x^2}}\right),$$

$$N(0,0) = [1+P(\sin^2\beta_0 - \cos^2\beta_0)]/4 = \frac{1}{4}\left(1 - P\frac{x}{\sqrt{1+x^2}}\right).$$

In the absence of any fields

$$\tau_{2S} = 2 \times 10^{-3} \text{ sec},$$

$$\tau_{2P} = 10^{-10} \text{ sec}.$$

The presence of an electric field shortens the lifetime of the metastable state of the Stark effect, which produces a mixing of the $2S_{1/2}$ and $2P_{1/2}$ states.

According to Lamb and Retherford:

$$\tau_S = \tau_P \left(\frac{\hbar^2(\omega^2 + \gamma^2/4)}{|V|^2} \right),$$

Where $\hbar\omega$ is the energy difference between the levels involved in the transition, $\gamma = 1/\tau_P$ ($\gamma/2\pi = 16$ GHz),

$$|V| = \int < \varphi_b | e\vec{E}\vec{r} | \varphi_a > \text{ dV}.$$

If an electric field is perpendicular to B, the allowed mixings: $\Delta m_J = \pm 1$, that is, $\alpha - f$ and $\beta - e$,

if E parallel to B, $\Delta m_J = 0$, the allowed transitions are $\alpha - e$ and $\beta - f$,

$\alpha - 2\mathrm{S}_{1/2}(m_J = +1/2)$,
$\beta - 2\mathrm{S}_{1/2}(m_J = -1/2)$,
$e - 2\mathrm{P}_{1/2}(m_J = +1/2)$,
$f - 2\mathrm{P}_{1/2}(m_J = -1/2)$.

$$|V| = \frac{\sqrt{3}}{2} a_0 \epsilon E \cos\omega t \approx 2.2 \times 10^{-18} E \cos\omega t,$$

where $a_0 = 0.529 \times 10^{-8}$ cm, $\epsilon \simeq 1$, ω – angular frequency of an oscillating electric field, equal zero for a static field, E (CGSE).

Populations of the α states:

$$N(\alpha) = N(1,1) + N(0,0) = \frac{1}{2}\left[1 + \frac{P}{2}\left(1 - \frac{x}{\sqrt{1+x^2}}\right)\right].$$

I_0 – zero polarization, I_+ – polarized beam, I_- – reversed polarization.

$$P = \frac{2}{1 - x/\sqrt{1+x^2}} \left(\frac{I_+}{I_0} - 1 \right).$$

$$P = \frac{2}{1 - x/\sqrt{1+x^2}} \left(\frac{I_+ - I_-}{I_+ + I_-} \right).$$

At the level crossing

$$\tau_s = \tau_P \frac{\hbar^2 \gamma^2}{4|V|^2}.$$

First level crossing ($\beta - e$) takes place at $B \approx 0.75$ T. In this case, the static electric field E should be perpendicular to the magnetic field B,

$\tau_\beta = 5.4 \times 10^{-5}/E^2$ s, E (V/cm),

$\tau_\alpha = 6.8 \times 10^{-2}/E^2$ s, ratio equals 1380.

Let a beam of metastable helium ions pass through a magnetic field (length$L$) corresponding the level crossing and in a rather weak electric field, so chosen that only small quantity of the ions in the α state decays, while practically all the ions in the β state are quenched to the ground state.

At $W = 20$ keV, $L = 3.4$ cm, $E = 90$ V/cm, 0.4% of the ions in the α state and 99the ions in the β state are quenched.

Second crossing ($\beta - f$) is at $B \approx 1.5$ T. Here E should be parallel B,

At $W = 20$ keV, $L = 1.2$ cm, $E = 150$ V/cm, $U = EL = 180$ V, the result is approximately the same.

Another possibility is to detect the atoms in α state using microwave quenching ($\nu = 9.35$ GHz, $\lambda = 3.2$ cm)

of β states at a relatively weak magnetic field 0.25 T.

In this case for $E_{\text{ampl.}} = 300$ V/cm at $\nu = 9.35$ GHz at $L = 1.2$ cm, 3% of the ions in the α state and 97% of the ions in the β state are quenched.

For final quenching (and measurement) of the atoms in the α state with transverse electric field E with $B = 0$, accepting $W = 20$ keV, $L = 3.4$ cm, $E = 90$ V/cm, 99% of the atoms in the α state are quenched.

Detecting 40.8 eV photons, we can measure nuclear polarization.

Additional quenching all the 2S states arises for off-axis beam particles in their passage through magnetic field which provides an effective electric field $\vec{E} = \frac{e}{c}\vec{v} \times \vec{B}$. In numeric calculations the magnetic field along the axis was accepted as linearly increasing up to maximum value at the length of 50 cm. The loss of the metastable beam has been estimated to be about 5% for a beam of diameter 6 mm and $B_{\max} = 0.75$ T.

There is also a loss of polarization due to radial components of the magnetic field an this has also been estimated $\leq 5\%$.

Correct consideration

$$i\hbar\frac{\partial\Psi}{\partial t} = (\hat{H} + \hat{H}')\Psi,$$

$\hat{H}$ is a time-independent Hamiltonian: $\hat{H}u_n = E_n u_n$.

Exact wave function is written in the form

$$\Psi = \sum a_n(t) u_n \mathrm{e}^{\frac{-iE_n t}{\hbar}}.$$

a_n must satisfy the equation

$$i\hbar\dot{a}_k = \sum H'_{kn} a_n \mathrm{e}^{i\omega_{kn}t},$$

where $\omega_{kn} = (E_k - E_n)/\hbar$, $H'_{kn} = \int u_k^* H' u_n \, \mathrm{d}\tau$.

$\Psi(t) = c_1(t)\phi^+_{\mathrm{He}}\phi^+_{\mathrm{e}} + c_2(t)\phi^-_{\mathrm{He}}\phi^+_{\mathrm{e}} + c_3(t)\phi^+_{\mathrm{He}}\phi^-_{\mathrm{e}} + c_4(t)\phi^-_{\mathrm{He}}\phi^-_{\mathrm{e}}$

$\phi^+_{\mathrm{He}}\phi^-_{\mathrm{e}} \rightarrow \cos\beta_0\psi(1,0)\exp[-i\omega(1,0)t] + \sin\beta_0\psi(0,0)\exp[-i\omega(0,0)t] \rightarrow [\cos\beta_0\cos\beta_1 + \sin\beta_0\sin\beta_1\exp(i\theta)]\phi^+_{\mathrm{He}}\phi^-_{\mathrm{e}} + [\cos\beta_0\sin\beta_1 - \sin\beta_0\cos\beta_1 \exp(i\theta)]\phi^-_{\mathrm{He}}\phi^+_{\mathrm{e}}$,

After averaging
$|c_2(t)|^2 = \cos^2\beta_0\sin^2\beta_1(t) + \sin^2\beta_0\cos^2\beta_1(t)$
$|c_3(t)|^2 = \cos^2\beta_0\cos^2\beta_1(t) + \sin^2\beta_0\sin^2\beta_1(t)$

$\phi^-_{\mathrm{He}}\phi^+_{\mathrm{e}} \rightarrow \sin\beta_0\psi(1,0)\exp[-i\omega(1,0)t] - \cos\beta_0\psi(0,0)\exp[-i\omega(0,0)t] \rightarrow [\sin\beta_0\cos\beta_1 - \cos\beta_0\sin\beta_1\exp(i\vartheta)]\phi^+_{\mathrm{He}}\phi^-_{\mathrm{e}} + [\sin\beta_0\sin\beta_1 + \cos\beta_0\cos\beta_1 \exp(i\vartheta)]\phi^-_{\mathrm{He}}\phi^+_{\mathrm{e}}$,

After averaging
$|c'_2|^2 = \cos^2\beta_0\cos^2\beta_1(t) + \sin^2\beta_0\sin^2\beta_1(t)$
$|c'_3|^2 = \cos^2\beta_0\sin^2\beta_1(t) + (\sin^2\beta_0\cos^2\beta_1(t)$

If $x \gg 1$, $\sin\beta_1 \to 0$, $\cos\beta_1 \to 1$,
$\phi^+_{\mathrm{He}}\phi^-_{\mathrm{e}} \to \cos\beta_0\phi^+_{\mathrm{He}}\phi^-_{\mathrm{e}} - \sin\beta_0\exp(i\theta)\phi^-_{\mathrm{He}}\phi^+_{\mathrm{e}}$
$\phi^-_{\mathrm{He}}\phi^+_{\mathrm{e}} \to \sin\beta_0\phi^+_{\mathrm{He}}\phi^-_{\mathrm{e}} + \cos\beta_0\exp(i\vartheta)\phi^-_{\mathrm{He}}\phi^+_{\mathrm{e}}$

$$N(\alpha) = \tfrac{1+P}{4} + \tfrac{1+P}{4}|c_2|^2 + \tfrac{1-P}{4}|c_2'|^2 =$$

$$\tfrac{1}{2} + \tfrac{P}{4}[1 + \cos^2\beta_0\sin^2\beta_1(t) + \sin^2\beta_0\cos^2\beta_1(t) -$$

$$\cos^2\beta_0\cos^2\beta_1(t) - \sin^2\beta_0\sin^2\beta_1(t)] \to$$

$$\tfrac{1}{2} + \tfrac{P}{4}(1 + \sin^2\beta_0 - \cos^2\beta_0) =$$

$$\tfrac{1}{2}\left[1 + \tfrac{P}{2}\left(1 - \tfrac{x}{\sqrt{1+x^2}}\right)\right].$$

For a high magnetic field ($x \gg 1$) the result is coincident with the simple consideration.

Conclusion

It seems feasible to make the polarimeter for $^3He^{++}$ beams with an energy of about 20 keV. It is possible to use a microwave field of 9.35 GHz at a magnetic field 0.25 T, or a static electric fields at 0.75 T, or 1.5 T. A systematic error of the polarization measurement estimated to be about 5%.