Spin structure of the „forward" charge-exchange reaction $n+p\to p+n$ and the deuteron charge-exchange breakup $d+p\to(pp)+n$

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1) Isotopic structure of NN-scattering

Operator describing the nucleon-nucleon scattering (taking into account the isotopic invariance):

$$\hat{f}(\vec{p},\vec{p}\,') = \hat{a}(\vec{p},\vec{p}\,') + \hat{b}(\vec{p},\vec{p}\,')\,\hat{\vec{\tau}}^{(1)}\hat{\vec{\tau}}^{(2)}$$

$\hat{\vec{\tau}}^{(1)}$, $\hat{\vec{\tau}}^{(2)}$ → vector Pauli operators in the isotopic space

$\hat{a}(\vec{p},\vec{p}\,')$, $\hat{b}(\vec{p},\vec{p}\,')$ → 4-row matrices in the spin space of two nucleons;

$\vec{p}$, $\vec{p}\,'$ → initial and final momenta in the c.m. frame;

directions of $\vec{p}\,'$ → defined within the solid angle in the c.m. frame, corresponding to the front hemisphere.

Process of elastic neutron-proton scattering into the back hemisphere → interpreted as the charge-exchange process $n+p \to p+n$.

Matrices of amplitudes of proton-proton, neutron-neutron and neutron-proton scattering:

$$\hat{f}_{pp\to pp}(\vec{p},\vec{p}') = \hat{f}_{nn\to nn}(\vec{p},\vec{p}') = \hat{a}(\vec{p},\vec{p}') + \hat{b}(\vec{p},\vec{p}')$$

$$\hat{f}_{np\to np}(\vec{p},\vec{p}') = \hat{a}(\vec{p},\vec{p}') - \hat{b}(\vec{p},\vec{p}')$$

Matrix of amplitudes of the charge transfer process:

$$\hat{f}_{np\to pn}(\vec{p},\vec{p}') = 2\hat{b}(\vec{p},\vec{p}') = \hat{f}_{pp\to pp}(\vec{p},\vec{p}') - \hat{f}_{np\to np}(\vec{p},\vec{p}')$$

States with total isotopic spins $T=1$ $(\hat{\vec{\tau}}^{(1)}\hat{\vec{\tau}}^{(2)}=1)$ and $T=0$ $(\hat{\vec{\tau}}^{(1)}\hat{\vec{\tau}}^{(2)}=-3)$ $\Longrightarrow$

matrices describing NN-scattering in the states with $T=1$ and $T=0$:

$$\begin{cases} \hat{f}^{(T=1)}(\vec{p},\vec{p}') = \hat{a}(\vec{p},\vec{p}') + \hat{b}(\vec{p},\vec{p}') \\ \hat{f}^{(T=0)}(\vec{p},\vec{p}') = \hat{a}(\vec{p},\vec{p}') - 3\hat{b}(\vec{p},\vec{p}') \end{cases} \Longrightarrow$$

$$\Rightarrow \begin{cases} \hat{a}(\vec{p},\vec{p}\,') = \frac{1}{4}\left(\hat{f}^{(T=0)}(\vec{p},\vec{p}\,') + 3\hat{f}^{(T=1)}(\vec{p},\vec{p}\,')\right) \\ \hat{b}(\vec{p},\vec{p}\,') = \frac{1}{4}\left(\hat{f}^{(T=1)}(\vec{p},\vec{p}\,') - \hat{f}^{(T=0)}(\vec{p},\vec{p}\,')\right) \end{cases}$$

$\Rightarrow$ we obtain in this representation:

$$\hat{f}_{pp\to pp}(\vec{p},\vec{p}\,') = \hat{f}_{nn\to nn}(\vec{p},\vec{p}\,') = \hat{f}^{(T=1)}(\vec{p},\vec{p}\,')$$

$$\hat{f}_{np\to np}(\vec{p},\vec{p}\,') = \frac{1}{2}\left(\hat{f}^{(T=1)}(\vec{p},\vec{p}\,') + \hat{f}^{(T=0)}(\vec{p},\vec{p}\,')\right)$$

$$\hat{f}_{np\to pn}(\vec{p},\vec{p}\,') = \frac{1}{2}\left(\hat{f}^{(T=1)}(\vec{p},\vec{p}\,') - \hat{f}^{(T=0)}(\vec{p},\vec{p}\,')\right)$$

Amplitude of the charge transfer reaction $\hat{f}_{np\to pn}(\vec{p},\vec{p}\,')$ defined in the front hemisphere $0 \leq \Theta \leq \frac{\pi}{2}$, $0 \leq \varphi \leq 2\pi$ ($\Theta \to$ angle between the momenta of initial neutron and final proton, $\varphi \to$ azimuthal angle) $\Rightarrow$ should be connected with the amplitude of neutron-proton scattering $\hat{f}_{np\to np}(\vec{p},-\vec{p}\,')$ in the back hemisphere by the angle $\tilde{\Theta} = \pi - \Theta$ at the azimuthal angle $\tilde{\varphi} = \pi + \varphi$ in the c.m. frame.

Relation between $\hat{f}_{np\to pn}(\vec{p},\vec{p}\,')$ and $\hat{f}_{np\to np}(\vec{p},-\vec{p}\,')$, following from the antisymmetry of the state of two fermions with respect to the total permutation, including the permutation of momenta $(\vec{p}\,'\to-\vec{p}\,')$, permutation of spin projections and permutation of isotopic projections $(p\rightleftarrows n)$:

$$\boxed{\begin{aligned}\hat{f}^{(T=1)}(\vec{p},\vec{p}\,') &= -\hat{P}^{(1,2)}\hat{f}^{(T=1)}(\vec{p},-\vec{p}\,'),\\ \hat{f}^{(T=0)}(\vec{p},\vec{p}\,') &= \hat{P}^{(1,2)}\hat{f}^{(T=0)}(\vec{p},-\vec{p}\,')\end{aligned}}$$

$$\to \hat{f}_{np\to pn}(\vec{p},\vec{p}\,') = -\hat{P}^{(1,2)}\hat{f}_{np\to np}(\vec{p},-\vec{p}\,'),$$

where $\hat{P}^{(1,2)}\to$ Hermitian and unitary operator of permutation of spin projections:

$$\hat{P}^{(1,2)} = \frac{1}{2}\left(\hat{I}^{(1,2)} + \hat{\vec{\sigma}}^{(1)}\hat{\vec{\sigma}}^{(2)}\right)$$

$\hat{I}^{(1,2)}\to$ unit 4-row matrix

$\hat{\vec{\sigma}}^{(1)}, \hat{\vec{\sigma}}^{(2)}\to$ vector Pauli operators

$$\langle m_1' m_2'|\hat{P}^{(1,2)}|m_1 m_2\rangle = \delta_{m_1' m_2}\,\delta_{m_2' m_1} \to \text{matrix elements.}$$

[V. L. Lyuboshitz, M. I. Podgoretsky, Yadernaya Fizika, 59 (3), 476 (1996) (Phys. At. Nucl. 59 (3), 449 (1996))]

In the case of the amplitude of the charge-exchange reaction $f_{np\to pn}(\vec{p},\vec{p}\,')$ → the two-row matrices with the index 1 act between the spin states of the initial neutron with the momentum $\vec{p}$ and the final proton with the momentum $\vec{p}\,'$; and the two-row matrices with the index 2 act between the spin states of the initial proton with the momentum $-\vec{p}$ and the final neutron with the momentum $(-\vec{p}\,')$.

In the case of the amplitude of elastic scattering into the back hemisphere $f_{np\to np}(\vec{p},-\vec{p}\,')$ → the matrices with the index 1 act between spin states of the neutrons with the momenta $\vec{p}$ and $(-\vec{p}\,')$, and the matrices with the index 2 act between spin states of the protons with the momenta $(-\vec{p})$ and $\vec{p}\,'$.

$\hat{P}^{(1,2)}\hat{P}^{(1,2)+} = \hat{P}^{(1,2)+}\hat{P}^{(1,2)} = \hat{I}^{(1,2)}$; hence → the matrix equality

$$\hat{f}^{+}_{np\to pn}(\vec{p},\vec{p}\,')\hat{f}_{np\to pn}(\vec{p},\vec{p}\,') = \hat{f}^{+}_{np\to np}(\vec{p},-\vec{p}\,')\hat{f}_{np\to np}(\vec{p},-\vec{p}\,').$$

As a result, at any polarizations of the initial nucleons the differential cross-sections of the exchange reaction $n+p\to p+n$ and the elastic np-scattering in the corresponding back hemisphere coincide:
$$\frac{d\sigma_{np\to pn}}{d\Omega}(\vec{p},\vec{p}\,') = \frac{d\sigma_{np\to np}}{d\Omega}(\vec{p},-\vec{p}\,').$$

(just as it should hold)

However, the separation into the spin-dependent and spin-independent parts is different for the amplitudes $\hat{f}_{np\to pn}(\vec{p},\vec{p}\,')$ and $\hat{f}_{np\to np}(\vec{p},-\vec{p}\,')$!

2) Nucleon charge-exchange process at zero angle.

Spin structure of the amplitude of the nucleon charge-exchange reaction in the „forward" direction in the c.m. frame of the (np)-system:

$$\hat{f}_{np\to pn}(0) = c_1 \hat{I}^{(1,2)} + c_2 \left[\hat{\vec{\sigma}}^{(1)}\hat{\vec{\sigma}}^{(2)} - (\hat{\vec{\sigma}}^{(1)}\vec{\ell})(\hat{\vec{\sigma}}^{(2)}\vec{\ell})\right] + \\ + c_3 (\hat{\vec{\sigma}}^{(1)}\vec{\ell})(\hat{\vec{\sigma}}^{(2)}\vec{\ell})$$

($\vec{\ell}$ – unit vector along the momentum of incident neutron).

Spin structure of the amplitude of elastic np-scattering in the „backward" direction:

$$\hat{f}_{np\to np}(\pi) = \tilde{c}_1 \hat{I}^{(1,2)} + \tilde{c}_2 \left[\hat{\vec{\sigma}}^{(1)}\hat{\vec{\sigma}}^{(2)} - (\hat{\vec{\sigma}}^{(1)}\vec{\ell})(\hat{\vec{\sigma}}^{(2)}\vec{\ell})\right] + \\ + \tilde{c}_3 (\hat{\vec{\sigma}}^{(1)}\vec{\ell})(\hat{\vec{\sigma}}^{(2)}\vec{\ell}).$$

Connection between the spin structure of the amplitudes $\hat{f}_{np\to pn}(0)$ and $\hat{f}_{np\to np}(\pi)$:

$\hat{f}_{np\to pn}(0) = -\hat{P}^{(1,2)}\hat{f}_{np\to np}(\pi)$; relations:

$$(\hat{\vec{\sigma}}^{(1)}\hat{\vec{\sigma}}^{(2)})(\hat{\vec{\sigma}}^{(1)}\hat{\vec{\sigma}}^{(2)}) = 3\hat{I}^{(1,2)} - 2\hat{\vec{\sigma}}^{(1)}\hat{\vec{\sigma}}^{(2)};$$

$$(\hat{\vec{\sigma}}^{(1)}\hat{\vec{\sigma}}^{(2)})(\hat{\vec{\sigma}}^{(1)}\vec{\ell})(\hat{\vec{\sigma}}^{(2)}\vec{\ell}) = \hat{I}^{(1,2)} - \hat{\vec{\sigma}}^{(1)}\hat{\vec{\sigma}}^{(2)} + (\hat{\vec{\sigma}}^{(1)}\vec{\ell})(\hat{\vec{\sigma}}^{(2)}\vec{\ell})$$

As a result:

$$\boxed{c_1 = -\frac{1}{2}(\tilde{c}_1 + 2\tilde{c}_2 + \tilde{c}_3);\quad c_2 = -\frac{1}{2}(\tilde{c}_1 - \tilde{c}_3);\quad c_3 = -\frac{1}{2}(\tilde{c}_1 - 2\tilde{c}_2 + \tilde{c}_3).}$$

Differential cross-sections (for unpolarized nucleons)

$$\frac{d\sigma_{np\to pn}}{d\Omega}(0) = |c_1|^2 + 2|c_2|^2 + |c_3|^2 = \frac{1}{4}|\tilde{c}_1 + 2\tilde{c}_2 + \tilde{c}_3|^2 +$$

$$+ \frac{1}{2}|\tilde{c}_1 - \tilde{c}_3|^2 + \frac{1}{4}|\tilde{c}_1 - 2\tilde{c}_2 + \tilde{c}_3|^2 = |\tilde{c}_1|^2 + 2|\tilde{c}_2|^2 + |\tilde{c}_3|^2 = \frac{d\sigma_{np\to np}}{d\Omega}(\pi)$$

Amplitudes of „forward" proton-proton and neutron-proton elastic scattering have the analogous spin structure => due to the isotopic invariance

$$c_1 = c_1^{(pp)} - c_1^{(np)};\quad c_2 = c_2^{(pp)} - c_2^{(np)};\quad c_3 = c_3^{(pp)} - c_3^{(np)}.$$

According to the optical theorem (using isotopic invariance)

$$\frac{4\pi}{k}\,\mathrm{Im}\, c_1 = \frac{4\pi}{k}\left(\mathrm{Im}\, c_1^{(pp)} - \mathrm{Im}\, c_1^{(np)}\right) = \sigma_{pp} - \sigma_{np} = \sigma_{nn} - \sigma_{np}$$

$\sigma_{pp}, \sigma_{nn}, \sigma_{np}$ → total cross-sections of interaction of two unpolarized protons (neutrons) and of an unpolarized neutron with an unpolarized proton, respectively.

$|\vec{p}| = k$ → neutron momentum in the c.m. frame of the colliding nucleons.

Differential cross-section of the process $n+p \to p+n$ in the „forward" direction for unpolarized nucleons:

$$\frac{d\sigma_{np\to pn}}{d\Omega}(0) = |c_1|^2 + 2|c_2|^2 + |c_3|^2 =$$

$$= \frac{d\sigma^{(si)}_{np\to pn}}{d\Omega}(0) + \frac{d\sigma^{(sd)}_{np\to pn}}{d\Omega}(0).$$

Here: $\frac{d\sigma^{(si)}_{np\to pn}}{d\Omega}(0) = |c_1|^2 = (\mathrm{Im}\, c_1)^2 + (\mathrm{Re}\, c_1)^2 =$

$$= \frac{k^2}{16\pi^2}(\sigma_{pp} - \sigma_{np})^2 (1+\alpha^2) \Rightarrow$$

$\left(\alpha = \frac{\mathrm{Re}\, c_1}{\mathrm{Im}\, c_1}\right) \Rightarrow$ spin-independent part of the cross-section of „forward" charge transfer process, determined by the difference of total cross-sections of unpolarized proton-proton and neutron-proton interaction.

$\frac{d\sigma^{(sd)}_{np\to pn}}{d\Omega}(0) = 2|c_2|^2 + |c_3|^2 \to$ spin-dependent part of the cross-section.

Transition to the differential cross-section $\left.\frac{d\sigma}{dt}\right|_{t=0}$ →
relativistic invariant $\left(t = -(p-p')^2 = (\vec{p}-\vec{p}')^2 - (E-E')^2 \rightarrow\right.$
→ square of 4-dimensional transferred momentum)

In the c.m. frame:

$$t = 2k^2(1-\cos\theta); \quad \frac{d\sigma}{d(-\cos\theta)} = 2\pi\frac{d\sigma}{d\Omega}; \quad \frac{d\sigma}{dt} = \frac{1}{2k^2}\frac{d\sigma}{d(-\cos\theta)} = \frac{\pi}{k^2}\frac{d\sigma}{d\Omega}$$

$$\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} = \left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} + \left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0};$$

$$\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} = \frac{\pi}{k^2}|c_1|^2; \quad \left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0} = \frac{\pi}{k^2}\left(2|c_2|^2 + |c_3|^2\right).$$

So, we obtain:

$$\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} = \left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0} + \frac{1}{16\pi}(\sigma_{pp}-\sigma_{np})^2(1+\alpha^2)$$

Impulse approach (the velocity of the initial neutron $v \gg \sqrt{\frac{\varepsilon_d}{m_d}} \sim 1/20$, ε_d - deuteron binding energy, m_d - deuteron mass, $\hbar = c = 1$) $\Longrightarrow$

there exists a simple connection between the spin-dependent part of the differential cross-section of the charge-exchange reaction $n+p \to p+n$ at zero angle $\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}$ (not the „backward" elastic neutron-proton scattering!) and the differential cross-section of the deuteron charge-exchange breakup $d+p \to (pp)+n$ in the „forward" direction $\left.\frac{d\sigma_{dp\to ppn}}{dt}\right|_{t=0}$ at the deuteron momentum $\vec{k}_d = 2\vec{k}_n$ (k_n - initial neutron momentum).

Due to the isotopic invariance $\Longrightarrow$ the same connection with the process $p+d \to n+(pp)$ at the proton momentum $\vec{k}_p = \vec{k}_n$ and with the process $n+d \to p+(nn)$ at the neutron laboratory momentum $\vec{k}_n$.

For unpolarized particles we have:

$$\left.\frac{d\sigma_{dp\to ppn}}{dt}\right|_{t=0} = \frac{2}{3}\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}$$

[N.W. Dean, Phys. Rev. D5, 1661 (1972)
Phys. Rev. D5, 2832 (1972)

V.V. Glagolev, V.L. Lyuboshitz, V.V. Lyuboshitz, N.M. Piskunov. JINR Communication E1-99-280, Dubna, 1999.

R. Lednicky, V.L. Lyuboshitz, V.V. Lyuboshitz, Proceedings of ISHEPP XVI, vol. I, JINR E1,2-2004-76, Dubna, 2004, p.199

This formula remains valid as well if not only the deuteron S-wave state, but also the deuteron D-wave state is taken into account.

The cross-section $\left.\frac{d\sigma_{dp\to ppn}}{dt}\right|_{t=0}$ is already integrated over the spectrum of relative momenta of two final protons.

[At zero transfer of momentum, two protons produced in the deuteron charge-exchange breakup prove to be in the states with even orbital momentum ($L=0$, or $L=0$ and $L=2$ when taking into account the D-wave). in the deuteron rest frame. It follows from the antisymmetry of the total wave function of two protons that they are in the singlet state (total spin $S=0$) at even orbital momenta $\Longrightarrow$ the transition from the triplet state of the neutron and proton in the deuteron into the singlet state of two protons may take place only due to the spin-dependent terms in the amplitude of the neutron-proton charge-exchange process $np \to pn$.]

So, finally the differential cross-section of the „forward" nucleon charge-exchange reaction takes the form:

$$\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} = \frac{3}{2}\left.\frac{d\sigma_{dp\to ppn}}{dt}\right|_{t=0} + \frac{1}{16\pi}\left(\sigma_{pp}-\sigma_{np}\right)^2\left(1+\alpha^2\right).$$

Thus, in principle, the modulus of the ratio of real and imaginary parts of the spin-independent charge transfer amplitude at zero angle ($|\alpha|$) may be determined using the experimental data on the total cross-sections of interaction of unpolarized nucleons and on the differential cross-sections of the processes $n+p \to p+n$ and $d+p \to (pp)+n$ in the „forward" direction.

At present there are no yet final reliable experimental data on the differential cross-section of the deuteron charge-exchange breakup in the „forward" direction. Analysis shows: if we suppose that the real part of the spin-independent amplitude of charge transfer $np \to pn$ at zero angle is smaller or of the same order as compared with the imaginary part

$(\alpha^2 \lesssim 1) \Longrightarrow$

then it follows from the available experimental data on the differential charge-exchange cross-section $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$ and the data on the total cross-sections σ_{pp} and σ_{np} that the main contribution into the cross-section $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$ at neutron laboratory kinetic energy $T_n \gtrsim 200\,MeV$ is provided namely by the spin-dependent part $\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}$.

[P. F. Shepard et al. Phys. Rev. D10, 2735 (1974)
T. J. Delvin et al. Phys. Rev. D8, 136 (1973)
J. L. Friedes et al. Phys. Rev. Lett. 15, 38 (1965)]

If the differential cross-section $\frac{d\sigma}{dt}$ is given in $mbn/\left(\frac{GeV}{c}\right)^2$ and the total cross-sections are given in mbn $\Rightarrow$ the spin-independent part of the charge-exchange cross-section

$$\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \approx 0.0512\left(\sigma_{pp}-\sigma_{np}\right)^2\left(1+\alpha^2\right)$$

$$\left(1\,mbn = 2.576\left(\frac{GeV}{c}\right)^{-2}\right)$$

Estimates of the ratio $\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \Big/ \left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$:

1) $k_n = 0.7\ \frac{GeV}{c}$; $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} = 268\ mbn\left(\frac{GeV}{c}\right)^{-2}$;

$\sigma_{pp}-\sigma_{np} = -22.6\ mbn \Rightarrow \left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \Big/ \left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} \approx 0.1(1+\alpha^2)$

2) $k_n = 1.7\ \frac{GeV}{c}$; $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} \approx 37.6\ mbn\left(\frac{GeV}{c}\right)^{-2}$;

$\sigma_{pp}-\sigma_{np} = 10\ mbn \Rightarrow \left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \Big/ \left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} \approx 0.136(1+\alpha^2)$

3) $k_n = 2.5\ \frac{GeV}{c}$; $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} \approx 17.85\ mbn\left(\frac{GeV}{c}\right)^{-2}$;

$\sigma_{pp}-\sigma_{np} = 5.5\ mbn \Rightarrow \left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \Big/ \left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} \approx 0.085(1+\alpha^2)$

Conclusion about the dominant role of the spin-dependent part of the amplitude of charge transfer $np\to pn$ at zero angle $\Rightarrow$ confirmed also by the preliminary data on the differential cross-section of „forward" deuteron charge-exchange breakup, obtained in Dubna (Laboratory of High Energies, JINR).

4 Summary

1. The structure of the nucleon charge-exchange process $n+p \to p+n$ is theoretically investigated on the basis of the isotopic invariance of the nucleon-nucleon scattering amplitude.
2. The nucleon charge-exchange reaction at zero angle is analyzed. Due to the optical theorem, the spin-independent part of the differential cross-section of „forward" nucleon charge-exchange reaction for unpolarized particles is connected with the difference of total cross-sections of unpolarized proton-proton and neutron-proton scattering.
3. The spin-dependent part of the differential cross-section of neutron-proton charge-exchange reaction at zero angle is proportional to the differential cross-section of „forward" deuteron charge-exchange breakup.

Analysis of the existing data shows that the main contribution into the differential cross-section of „forward" nucleon charge-exchange reaction is provided namely by the spin-dependent part.