J.G. Körner at the XII Workshop on High Energy Spin Physics

DUBNA-SPIN-07, Dubna, September 3 - 7, 2007

Gluon cut dependence of unpolarized and polarized structure functions in $e^+e^- \to Q(\uparrow)\bar{Q}(G)$

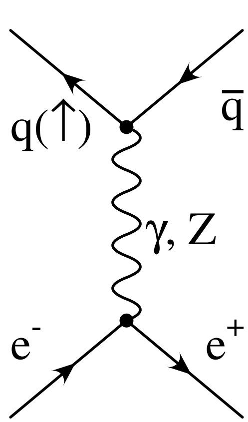
In collaboration with S. Groote (University of Tartu, Estonia)

For the planned International Linear Collider (ILC) one needs NLO corrections to unpolarized and polarized structure functions in top quark pair production $e^+e^- \rightarrow t(\uparrow)\bar{t}$.

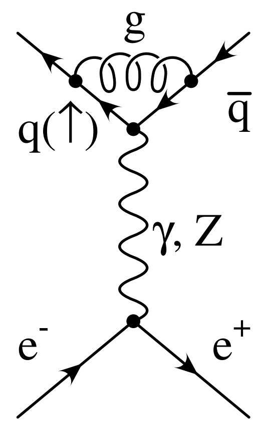
It would be very useful to partition the NLO corrections into

- NLO corrections in the soft gluon region (tests of QCD; comparison with non-SM physics)
- NLO corrections in the hard gluon region (additional tests of QCD and comparison with non-SM physics)

The two regions are separated by a cut on the energy of the gluon. It is best to have the results in the two regions in analytical form. This is the subject of my talk.



Born term



one–loop amplitude

tree diagrams

First on the list of agenda is to enumerate the number of structure functions

Twelve Invariant Structure Functions

$$
\begin{aligned}
H_{\mu\nu} = \quad & - \quad g_{\mu\nu} H_1^{pc} + p_{1\mu} p_{1\nu} H_2^{pc} + i\epsilon(\mu\nu p_1 q) H_3^{pv} \\
& + \quad (q \cdot s) \; \left[-g_{\mu\nu} G_1^{pv} + p_{1\mu} p_{1\nu} G_2^{pv} + i\epsilon(\mu\nu p_1 q) G_3^{pc} \right] \\
& + \quad (s_\mu p_{1\nu} + s_\nu p_{1\mu}) G_6^{pv} + \big(p_{1\mu} i\epsilon(\nu q p_1 s) - p_{1\nu} i\epsilon(\mu q p_1 s) \big) G_7^{pc} \\
& + \quad i\epsilon(\mu\nu q s) G_8^{pc} + i\epsilon(\mu\nu p_1 s) G_9^{pc} \\
& + \quad (s_\mu p_{1\nu} - s_\nu p_{1\mu}) G_{10}^{pv} + \Bigg(p_{1\mu} i\epsilon(\nu q p_1 s) + p_{1\nu} i\epsilon(\mu q p_1 s) \Bigg) G_{12}^{pc}
\end{aligned}
$$

- $H_1^{pc}, H_2^{pc}, H_3^{pv}$ unpolarized
- $G_1^{pv}, G_2^{pv}, G_3^{pc}, G_6^{pv}, G_7^{pc}, G_8^{pc}, G_9^{pc}$ polarized; T-even
- G_{10}^{pv}, G_{12}^{pc} polarized; T-odd (imaginary part of 1-loop)

Ten Helicity Structure Functions

- $H_U^{pc}, H_L^{pc}, H_F^{pv}$ unpolarized
- $H_{U^l}^{pv}, H_{L^l}^{pv}, H_{F^l}^{pc}, H_{I^\perp}^{pv}, H_{A^\perp}^{pc}$ polarized; T-even
- $H_{A^N}^{pv},\ H_{I^N}^{pc}$ polarized; T-odd (imaginary part of 1-loop)

Invariant structure functions are overcounted because of two nontrivial identities in four space-time dimensions due to Schouten's identity :

$$
\begin{aligned}
(p_1 q)\, i\epsilon(\mu\nu q s) &= -(qs)\, i\epsilon(\mu\nu p_1 q) + q^2 i\epsilon(\mu\nu p_1 s) \\
m_t^2 i\epsilon(\mu\nu q s) &= \Big(p_{1\mu} i\epsilon(\nu q p_1 s) - p_{1\nu} i\epsilon(\mu q p_1 s) \Big)
\end{aligned}
$$

Differential unpolarized and polarized rate distributions

unpolarized differential rate distribution : $\left(\sigma := \sigma_U + \sigma_L\right)$

$$\frac{d\sigma}{d\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\,\sigma_U + \frac{3}{4}\sin^2\theta\,\sigma_L + \frac{3}{4}\cos\theta\,\sigma_F$$

longitudinally polarized rate: $\left(\sigma^l_L = 0 \text{ at LO}\right)$

$$\frac{d\sigma^l}{d\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\,\sigma^l_U + \frac{3}{4}\sin^2\theta\,\sigma^l_L + \frac{3}{4}\cos\theta\,\sigma^l_F$$

transversely polarized rate:

$$\frac{d\sigma^\perp}{d\cos\theta} = -\frac{3}{\sqrt{2}}\sin\theta\cos\theta\,\sigma^\perp_I - \frac{3}{\sqrt{2}}\sin\theta\,\sigma^\perp_A$$

normally polarized rate (normal defined w.r.t. beam plane):

$$\frac{d\sigma^N}{d\cos\theta} = -\frac{3}{\sqrt{2}}\sin\theta\cos\theta\,\sigma^N_I - \frac{3}{\sqrt{2}}\sin\theta\,\sigma^N_A$$

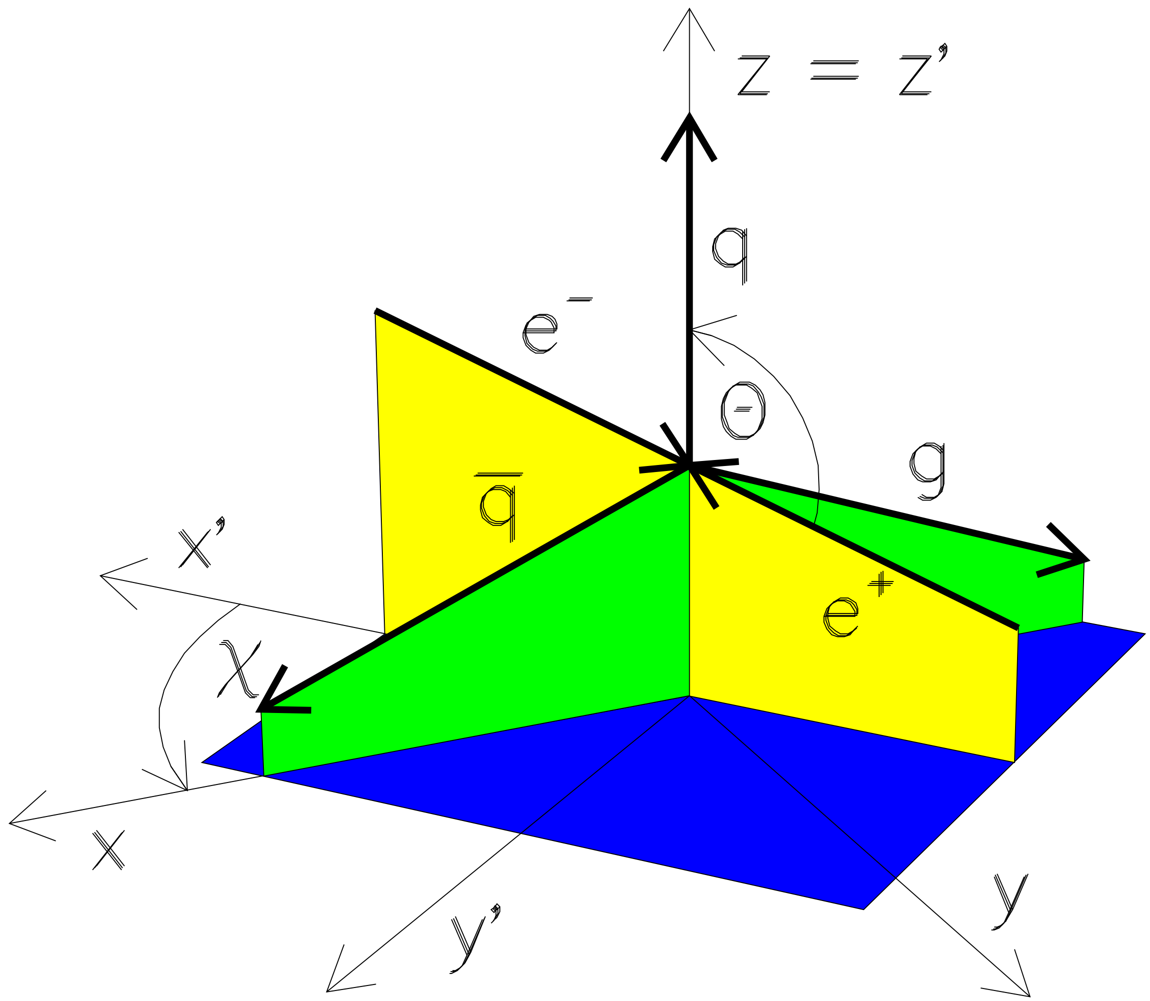


Figure 1: Definition of polar angle θ

Some technical details on the calculation. Most difficult is the tree graph integration up to a given gluon energy cut.

- Three-particle final state $\longrightarrow$ two-dimensional phase space integration with a cut on the gluon energy ($\lambda = E_G^{cut}/\sqrt{q^2}$)
- there are three mass scales in the problem: $\sqrt{q^2}, m_t, E_G^{cut}$ $\longrightarrow$ many different square roots, logs, dilogs ....in the final result
- infrared singularity regularized with a (small) gluon mass m_G
- For $\lambda \to \lambda_{\rm max}$ we obtain the cut independent known NLO results
  S. Groote, A. Pilaftsis, M.M. Tung, JGK (96, 97)
  W. Ravindran, W.L. van Neerven (2000) and others
- For the tree graph contribution we took the limit $\lambda \to \lambda_{\rm min}$ and obtained the usual soft gluon expressions
  $\sigma_i^{(m)}(soft; \lambda \to \lambda_{\rm min}) \to \sigma_i^{(m)}(\mathrm{Born})$ x soft-gluon-factor

- Comparison of cut–dependent unpolarized structure functions
  $\sigma_U, \sigma_L, \sigma_F$ with
  A.B. Arbuzov, D.Y. Bardin, A. Leike (92)
  J.B. Stav, H.A. Olsen (96)

Closed form expressions for $\sigma_i^{(m)}(soft;\lambda)$ are too long to be shown in this talk. The corresponding hard rates $\sigma_i^{(m)}(hard;\lambda)$ can be obtained by subtraction, i.e. $\sigma(hard) = \sigma - \sigma(soft)$.
$(i = U, L, F, I, A;\ m = 0, l, \perp, N)$

$\sigma_i^{(m)}$: $\sigma_U, \sigma_L, \sigma_F, \sigma_U^l, \sigma_L^l, \sigma_F^l, \sigma_I^\perp, \sigma_A^\perp, \sigma_I^N, \sigma_A^N$

Eikonal and soft gluon approximation

Eikonal approximation:

$$H^i_{\mu\nu}(eik) = \underbrace{g_s^2 C_F \left(\frac{p_1^2}{(p_1p_3)^2} - \frac{2(p_1p_2)}{(p_1p_3)(p_2p_3)} + \frac{p_2^2}{(p_2p_3)^2} \right)}_{\text{eikonal amplitude } A_{\rm eik}} H^i_{\mu\nu}(Born)$$

Integrate the eikonal amplitude $A_{\rm eik}$ up to a given gluon energy cut $E_G^{cut} = \lambda\sqrt{q^2}$:

The result is

$$
\begin{aligned}
h_{\mathrm{eik}} \quad = \quad & -\frac{\alpha_s C_F}{\pi v}\Bigg\{ \left(2v-(2-\xi)\ln\left(\frac{1+v}{1-v}\right)\right)\ln\left(\frac{2\lambda}{\sqrt{\Lambda}}\right)+4\left(\sqrt{1-2\lambda}\sqrt{1-2\lambda-\xi}-v\right) \\
& +2v\left(\ln\left(\frac{z_\lambda}{z_0}\right)+2\ln\left(\frac{z_0^2-1}{z_\lambda z_0-1}\right)\right)-\ln z_0+4\lambda\ln z_\lambda \ + \\
& +(2-\xi)\left(\frac{1}{2}\ln^2\left(\frac{z_\lambda}{z_0}\right)+2\ln z_0\ln\left(\frac{z_\lambda z_0-1}{z_0^2-1}\right)+\frac{1}{4}\ln^2 z_0 \ + \right. \\
& \left. +\mathrm{Li}_2\left(\frac{2v}{1+v}\right)+\mathrm{Li}_2\left(1-\frac{z_\lambda}{z_0}\right)+\mathrm{Li}_2(1-z_\lambda z_0)-\mathrm{Li}_2(1-z_0^2)\right)\Bigg\}
\end{aligned}
$$

where

$$
z_0=\frac{1+v}{1-v}, \qquad z_\lambda=\frac{\sqrt{1-2\lambda}+\sqrt{1-2\lambda-\xi}}{\sqrt{1-2\lambda}-\sqrt{1-2\lambda-\xi}}.
$$

λ is scaled gluon energy cut, Λ is scaled gluon mass ($\Lambda = m_G/\sqrt{q^2}$)

$\lambda \to 0$ limit gives the usual soft gluon factor:

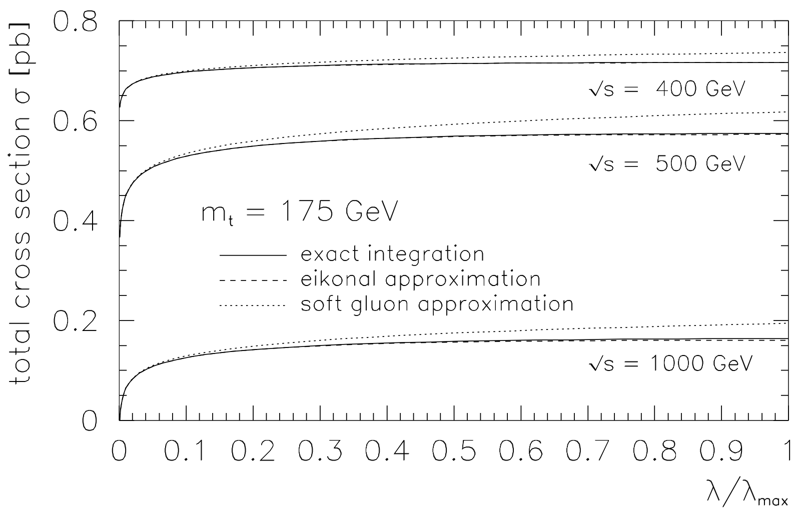
$$h_{\text{SGA}} = -\frac{\alpha_s C_F}{\pi v} \Bigg\{ \left(2v + (2-\xi) \ln\left(\frac{1-v}{1+v}\right) \right) \ln\left(\frac{2\lambda}{\sqrt{\Lambda}}\right) + \ln\frac{1-v}{1+v} + (2-\xi)\left(\frac{1}{4}\ln^2\left(\frac{1-v}{1+v}\right) + \text{Li}_2\left(\frac{2v}{1+v}\right) \right) \Bigg\}$$

Infrared (IR) singular piece

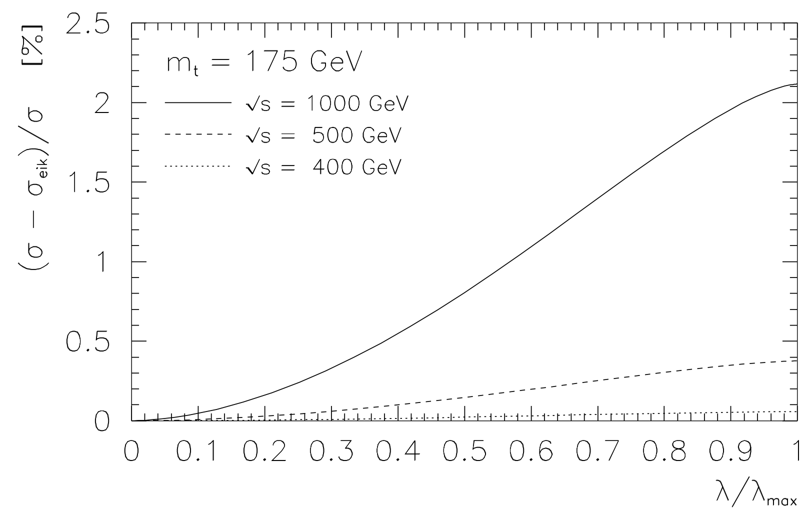
$$h_{IR} = -\frac{\alpha_s C_F}{\pi v} \Bigg\{ \left(2v + (2-\xi) \ln\left(\frac{1-v}{1+v}\right) \right) \ln\frac{1}{\sqrt{\Lambda}} \Bigg\}.$$

cancels against the corresponding loop contribution. The remaining IR finite pieces are then $h'_{\text{eik}} = h_{\text{eik}} - h_{IR}$ and $h'_{\text{SGA}} = h_{\text{SGA}} - h_{IR}$.

The eikonal approximation is an excellent approximation to the exact NLO result for the total rate $\sigma \equiv \sigma_{U+L}$ even in the hard gluon region:



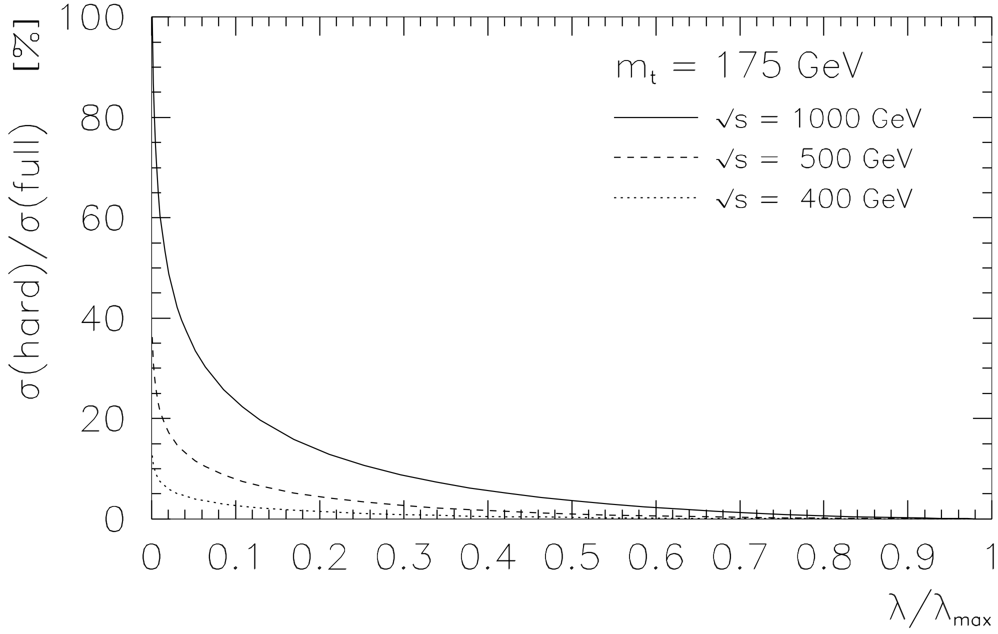
total cross section: exact and eikonal approximation



quality of eikonal approximation

Some numerical examples:

Hard gluon rate



NLO Polarization–type observables

$$P_i^{(m)}(NLO) = \frac{\sigma_i^{(m)}(Born) + \sigma_i^{(m)}(\alpha_s)}{\sigma(Born) + \sigma(\alpha_s)}$$

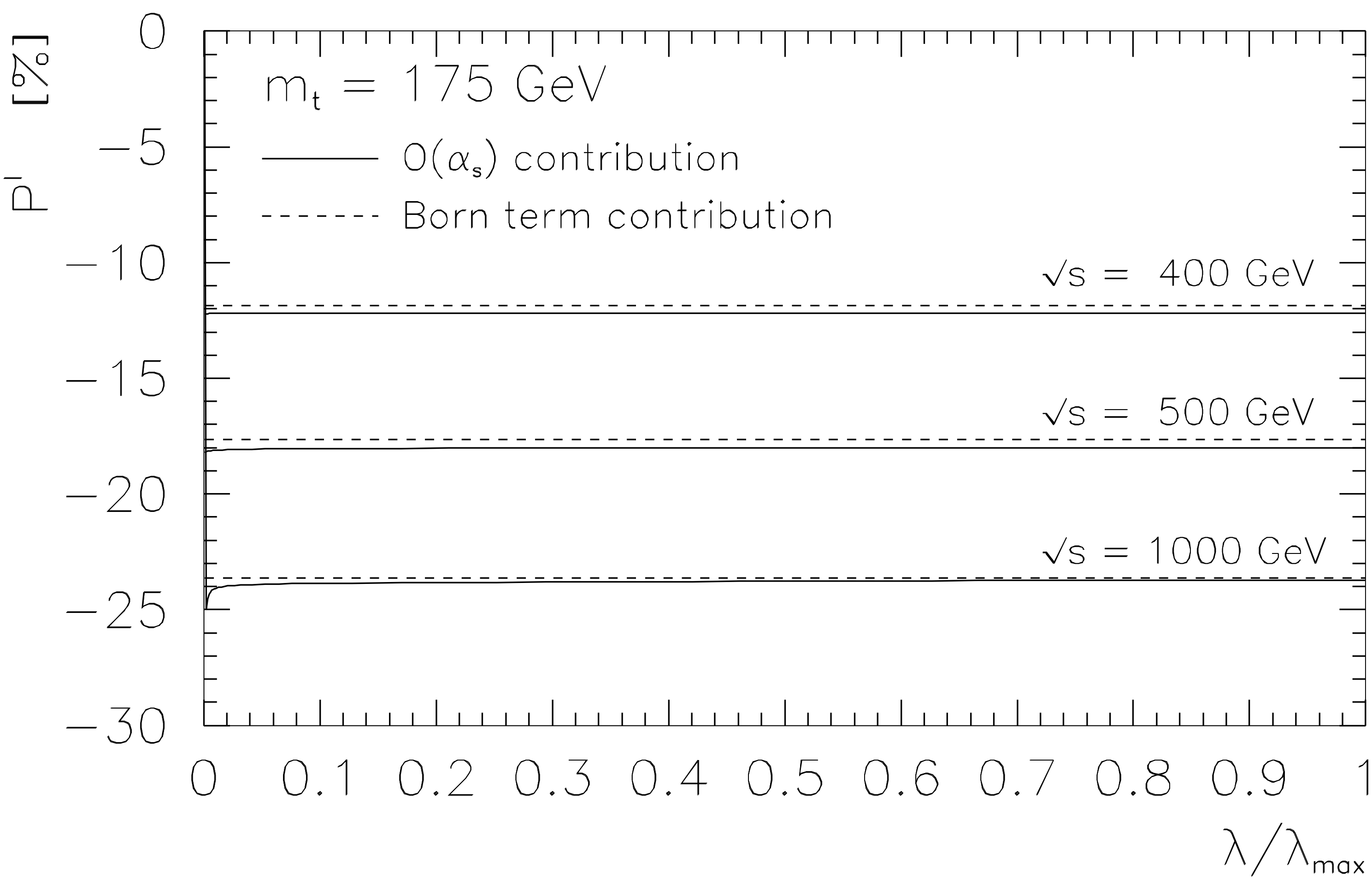
$$\approx \frac{\sigma_i^{(m)}(Born)(1 + h'_{\rm eik}(\alpha_s))}{\sigma(Born)(1 + h'_{\rm eik}(\alpha_s))} = P_i^{(m)}(Born)$$

approximately ($\approx$) means

- neglecting non-Born like hard tree contributions
- neglecting non-Born like loop contributions

$P_i^{(m)}(NLO) \approx P_i^{(m)}(Born)$ good to $O(1\% - 3\%)$ depending on the polarization observable and on the value of the cut parameter λ.

Example: Longitudinal polarization of top quark $P^l = \sigma^l / \sigma$

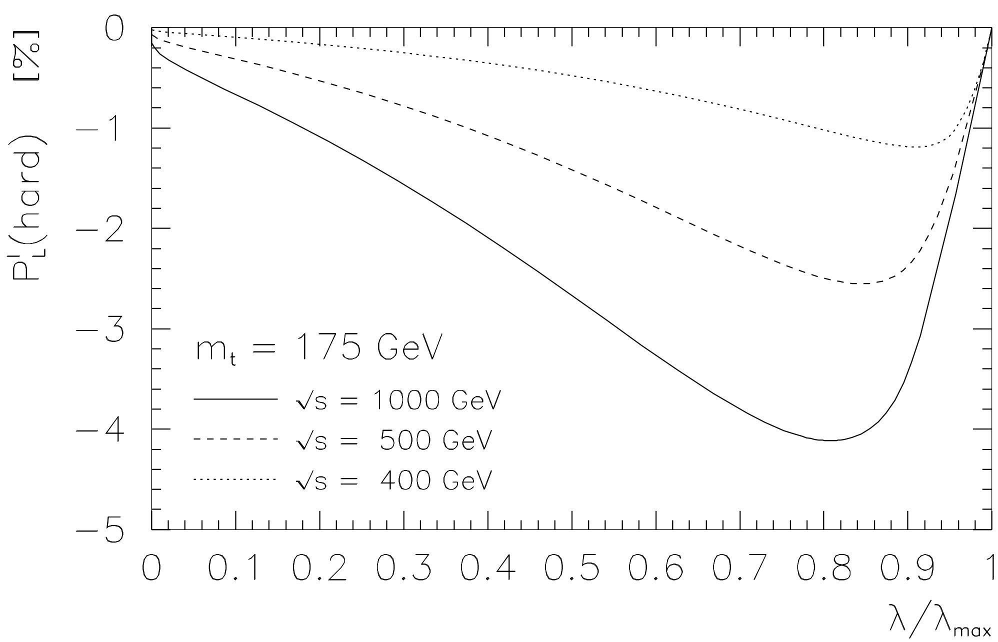


NLO Polarization–type observables in the hard region

$$P_i^{(m)}(NLO) = \frac{\sigma_i^{(m)}(\alpha_s; hard)}{\sigma(\alpha_s; hard)}$$

$P_i^{(m)}(NLO)$ deviates from $P_i^{(m)}(Born)$ by approximately $O(5\% - 10\%)$ depending on the polarization observable and on the value of the cut parameter λ.

Example: Longitudinal polarization from a longitudinally polarized initial gauge boson (γ^*, Z^*): P_L^l. This polarized structure function is interesting since $P_L^l(Born, one-loop) = 0$ (due to the absence of second class currents in the SM *and* having a two-body final state, i.e. also no one-loop contribution).



Summary and Conclusion

- We calculated the NLO corrections to three unpolarized structure functions and seven polarized structure functions up to a given gluon energy cut.
- We checked our calculations by taking the soft and hard gluon limits of our results which afforded a comparison with known results
- Polarization-type observables show little dependence on the cut parameter up to high values of the cut parameter. The deviation from the LO Born term results is small $O(1\% - 3\%)$ depending on the polarization observable and on the value of the cut parameter λ.

- In the hard gluon region the polarization-type observables deviate from their Born term values by $O(3\% - 10\%)$ depending on the polarization observable and on the value of the cut parameter λ.